## EXAM RULES

1) One 8.5" x 11" sheet permitted;
otherwise closed books, closed notes, open minds!

2) Scientific calculator is permitted; no computers and no phone!

3) **NO PHONES !!!**
Phones should not be visible at all!
A visible phone will result in a zero grade for the exam!

4) Write FULL NAME legibly on each page provided.

5) **PLEASE SHOW ALL WORK !!!**
**This is essential to receive partial credit!**

6) Please do not submit multiple answers. You must pick an answer!!

7) Write your solutions on the sheets provided.
No other paper/sheets/pages should be used!

8) Clearly label voltages and currents on the circuits provided.

9) Use the variables provided!
No additional variables should be used!

10) Unreadable work will receive NO CREDIT.

11) Please place important equations and answers within boxes as we have done in lecture.

12) Please be careful with your algebra, signs, etc.

13) Please turn in your solutions to me at the end of the period!

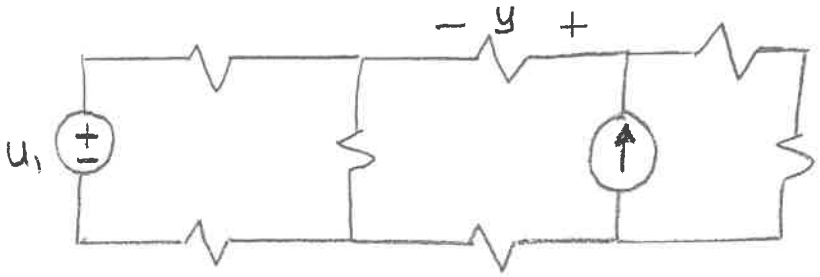
14) **PLEASE DO NOT CHEAT !!!**

*From the desk of Dr. A.A. Rodriguez*

# Problem #1

Relate y to $u_1, u_2$

(All R = 1)



# Problem #2

Relate $w_1, w_2, w_3, w_4$ to $u_1, u_2$

(Just set up the equations!)

(Nodal Analysis)

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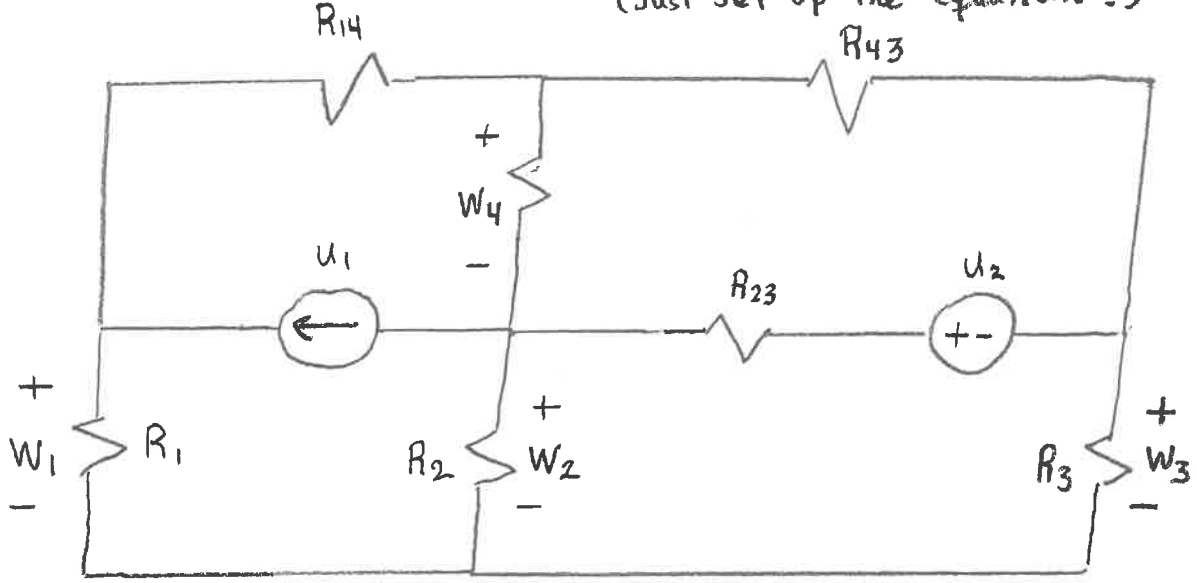

# Problem #3

Relate $I_1, I_2, I_3, I_4$ to $u_1, u_2, u_3, u_4$

(Just set up the equations !)

(Mesh Analysis)

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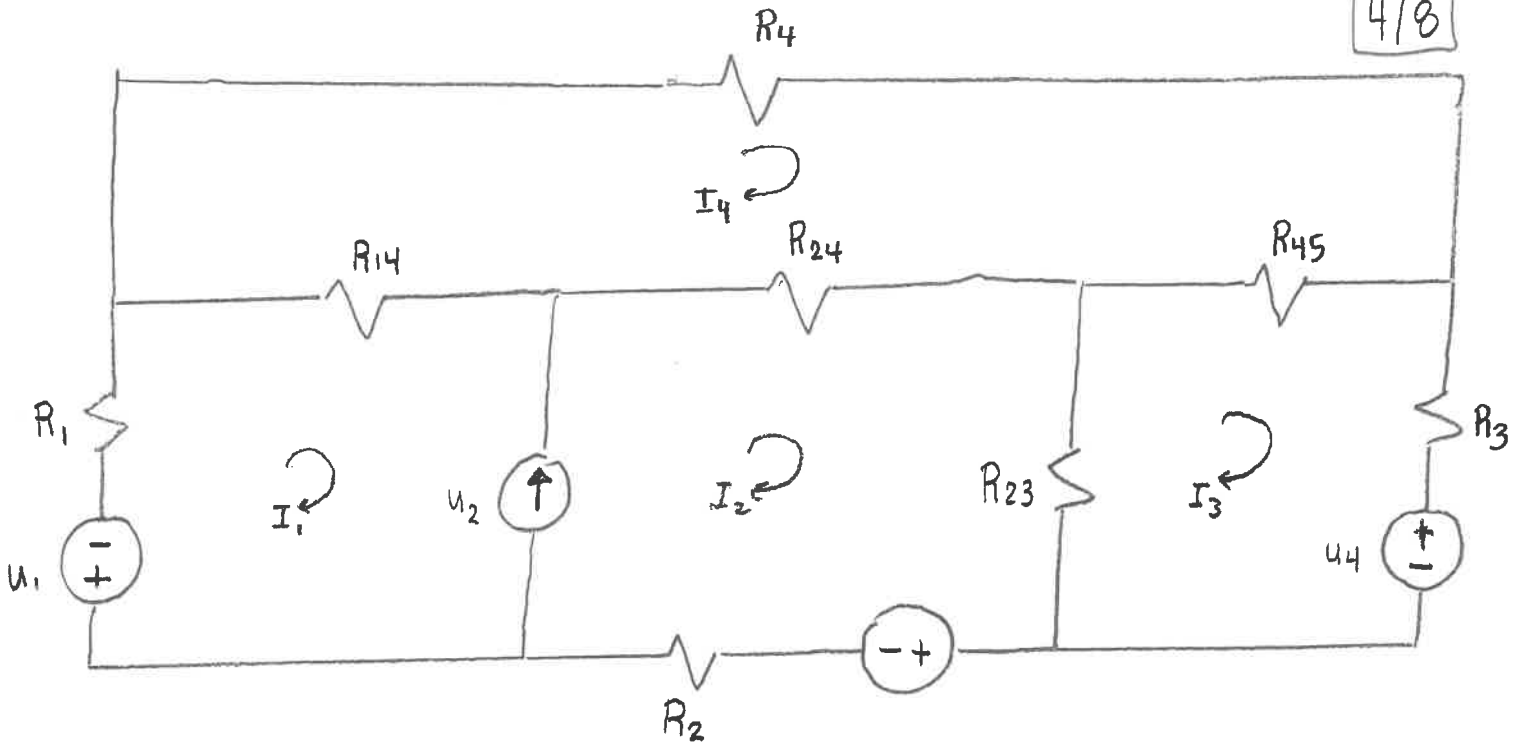
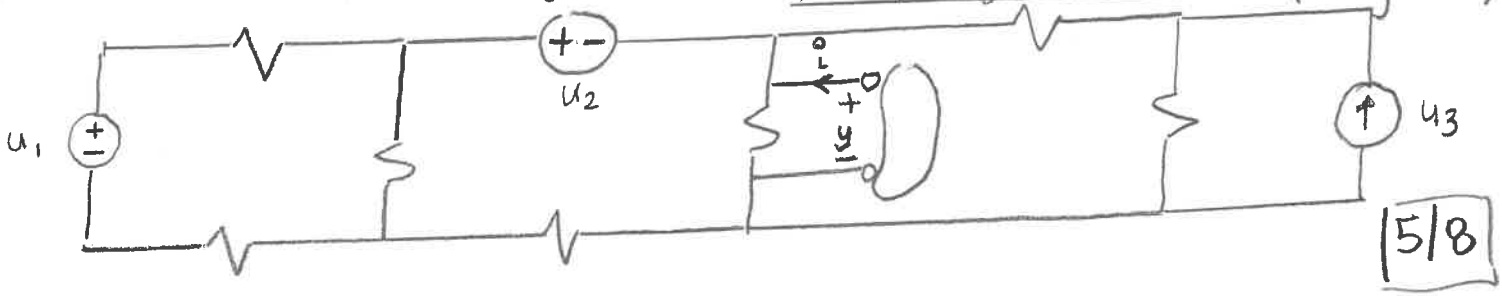


# Problem #4

Find a thevenin & Norton Equivalent at y (Thevenin-Norton at y)

(All R = 1) ... Must specify Rth & Vth

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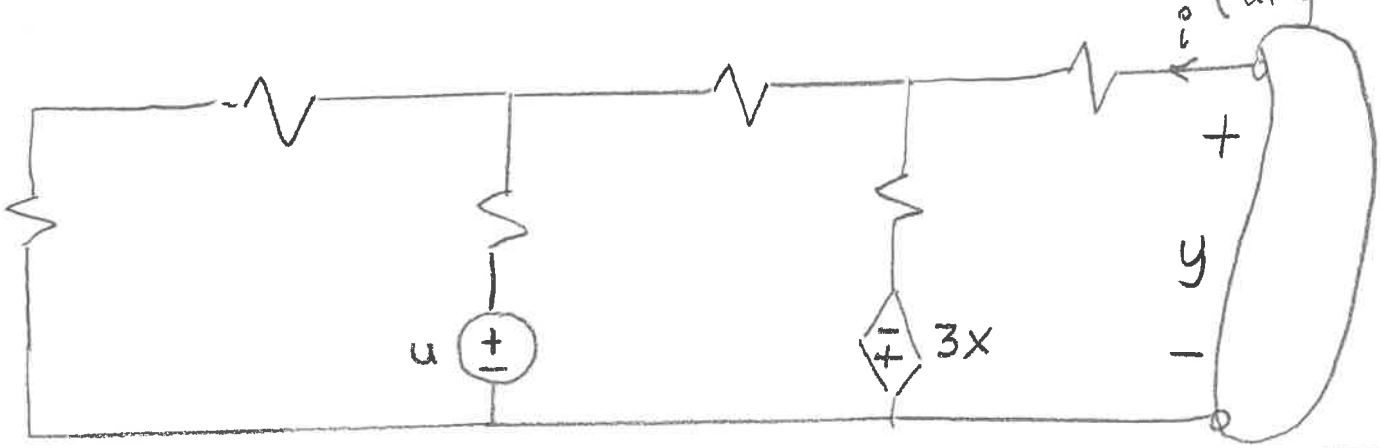
# Problem # 5

Find a Thevenin & Norton Equivalent at y

(All R = 1) --- Must specify $R_{th}$ & $V_{th}$

(Thevenin-Norton at y)

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# Problem #6

Find a Thevenin & Norton Equivalent at y

(All R=1) ... must specify $R_{th}$ & $V_{th}$

(Thevenin-Norton at y)

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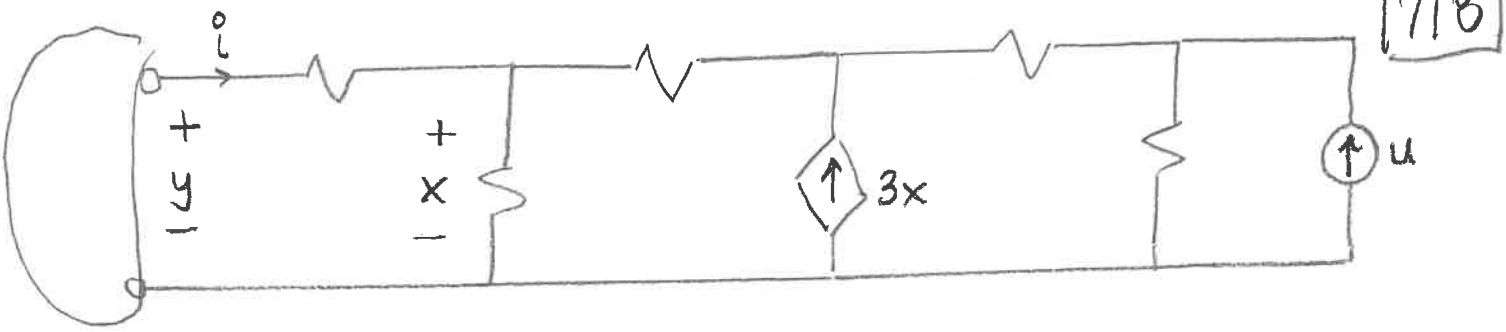

# Problem #7

Relate $y$ to $u_1, u_2$ (All $R = 1$) (Need Another Variable)

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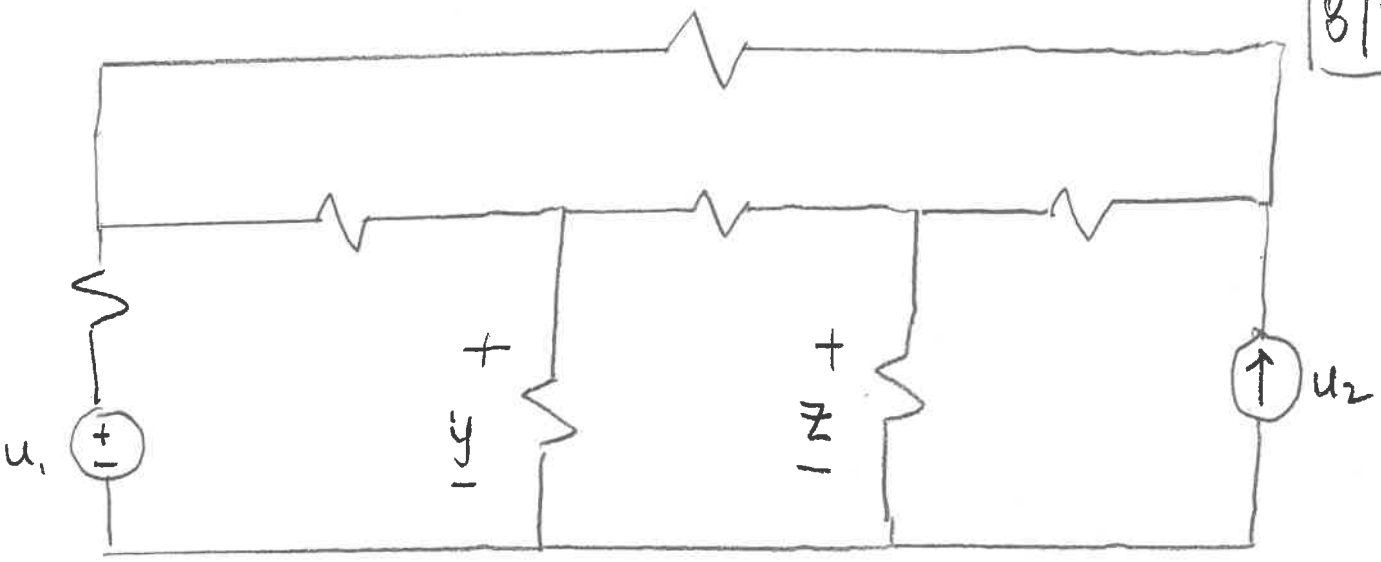
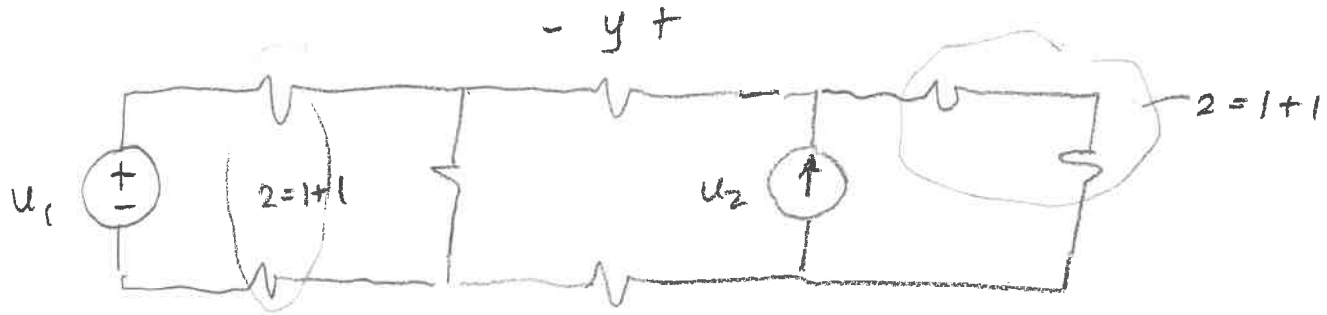
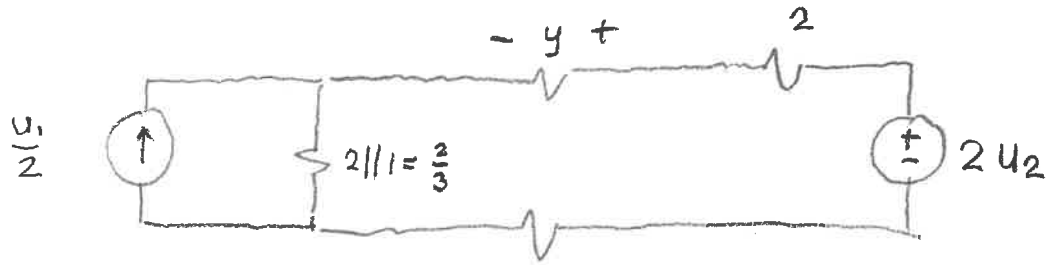


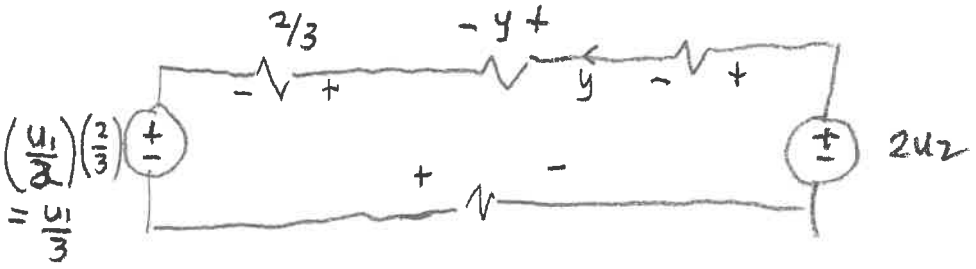
## Problem 1




Relate y to $u_{1,2}$ (All R's = 1)

[a] sol via source transf




$$\frac{u_1}{3} = -\left(\frac{2}{3}\right)y - y - y + 2u_2 - y = -3\tfrac{2}{3}y + 2u_2$$

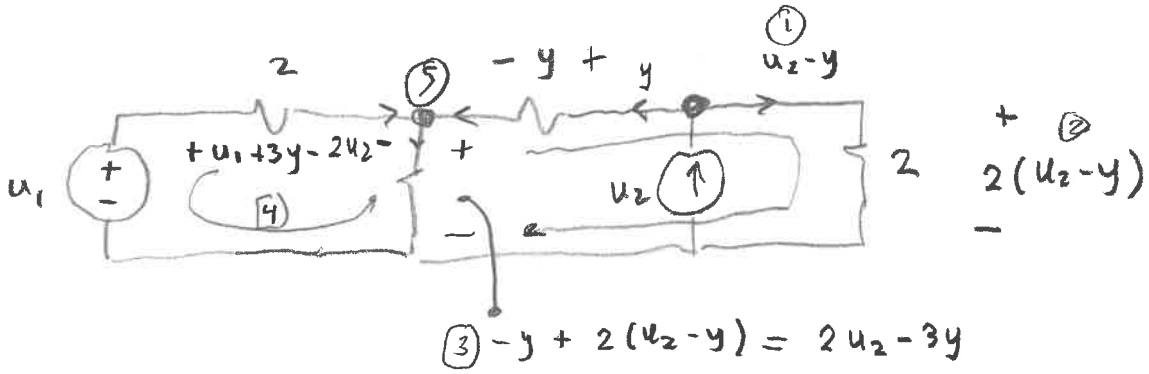
$$\frac{11}{3}y = -\frac{u_1}{3} + 2u_2$$

$$11y = -u_1 + 6u_2$$

$$\boxed{y = -\left(\frac{1}{11}\right)u_1 + \left(\frac{6}{11}\right)u_2}$$

# Problem 1

[b] Sol via KVL, KCL, Ohm (All R's = 1)




[5] KCL = $\frac{(u_1 + 3y - 2u_2)}{2} + y = 2u_2 - 3y$

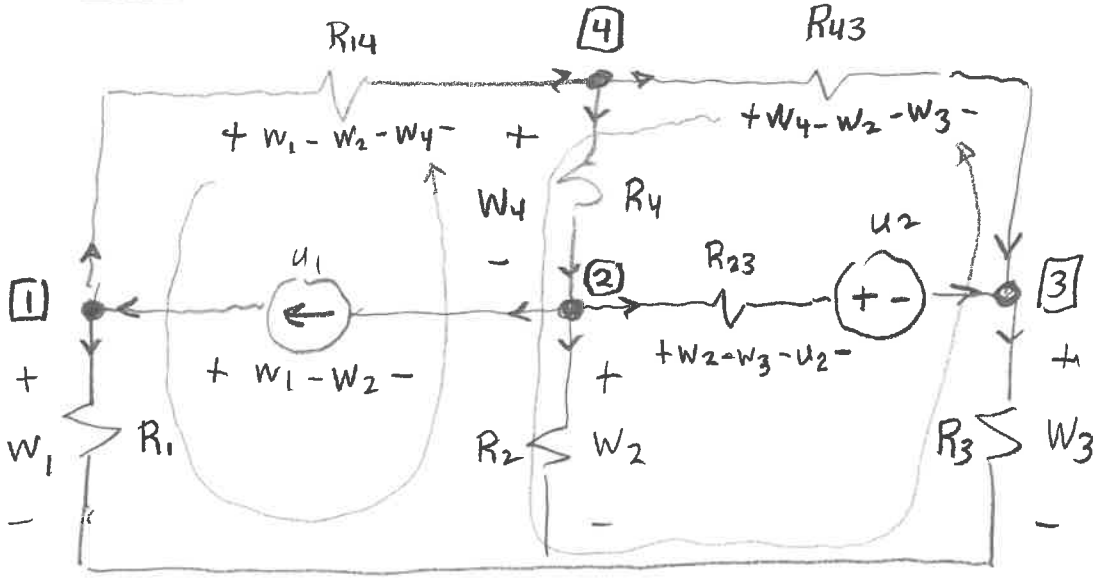
$u_1 + 3y - 2u_2 + 2y = 4u_2 - 6y$

$11y = -u_1 + 6u_2$

$y = -\left(\frac{1}{11}\right)u_1 + \left(\frac{6}{11}\right)u_2$

# Problem 2 (Nodal Analysis)

Relate $w_{1,2,3,4}$ to $u_{1,2}$ (Just set up the equations!)




1 KCL = $\overleftarrow{u_1} = \left(\frac{w_1}{R_1}\right) + \left(\frac{w_1 - w_2 - w_4}{R_{14}}\right)$

2 KCL = $\left(\frac{\downarrow w_4}{R_4}\right) = \overleftarrow{u_1} + \left(\frac{\downarrow w_2}{R_2}\right) + \left(\frac{\rightarrow w_2 - w_3 - w_2}{R_{23}}\right)$

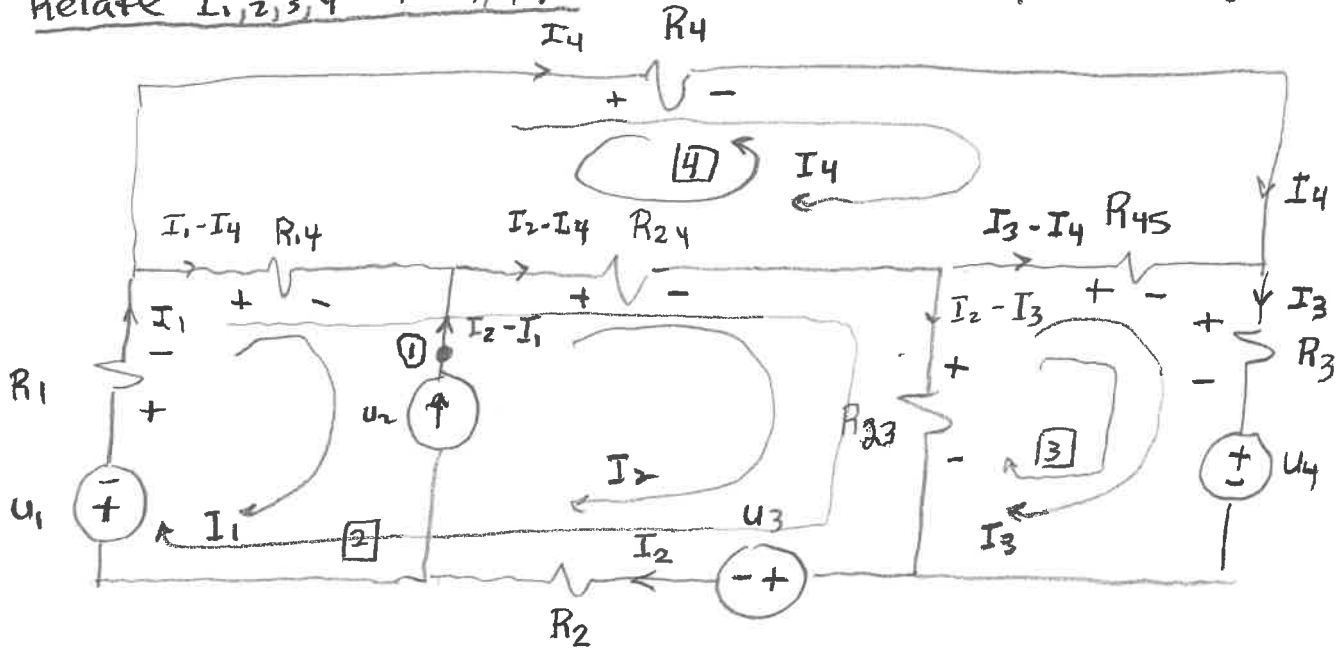
3 KCL = $\left(\frac{\rightarrow w_2 - w_3 - u_2}{R_{23}}\right) + \left(\frac{\downarrow w_4 - w_2 - w_3}{R_{43}}\right) = \left(\frac{\downarrow w_3}{R_3}\right)$

4 KCL = $\left(\frac{\rightarrow w_1 - w_2 - w_4}{R_{14}}\right) = \left(\frac{\downarrow w_4}{R_4}\right) + \left(\frac{\rightarrow w_4 - w_2 - w_3}{R_{43}}\right)$

$\Rightarrow$ 4 eqs in 4 unknowns $w_{1,2,3,4}$ ! :o

# Problem 3 (Mesh Analysis)

Relate $I_{1,2,3,4}$ to $u_{1,2,3,4}$ (Just set up the equations!)




[1] KCL = $I_2 - I_1 = u_2$

[2] KVL Bottom 2 left loops

$$-R_1 I_1 - u_1 = R_{14}(I_1 - I_4) + R_{24}(I_2 - I_4) + R_{23}(I_2 - I_3) + u_3 + R_2 I_2$$

[3] KVL = bottom right loop

$$R_{23}(I_2 - I_3) = R_{45}(I_3 - I_4) + R_3 I_3 + u_4$$

[4] KVL = top loop

$$R_4 I_4 = R_{14}(I_1 - I_4) + R_{24}(I_2 - I_4) + R_{45}(I_3 - I_4)$$

$\Rightarrow$ 4 eqs in 4 unknowns $I_{1,2,3,4}$

# Problem 4 (Norton & Thevenin Equivalent)

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(All R's = 1)

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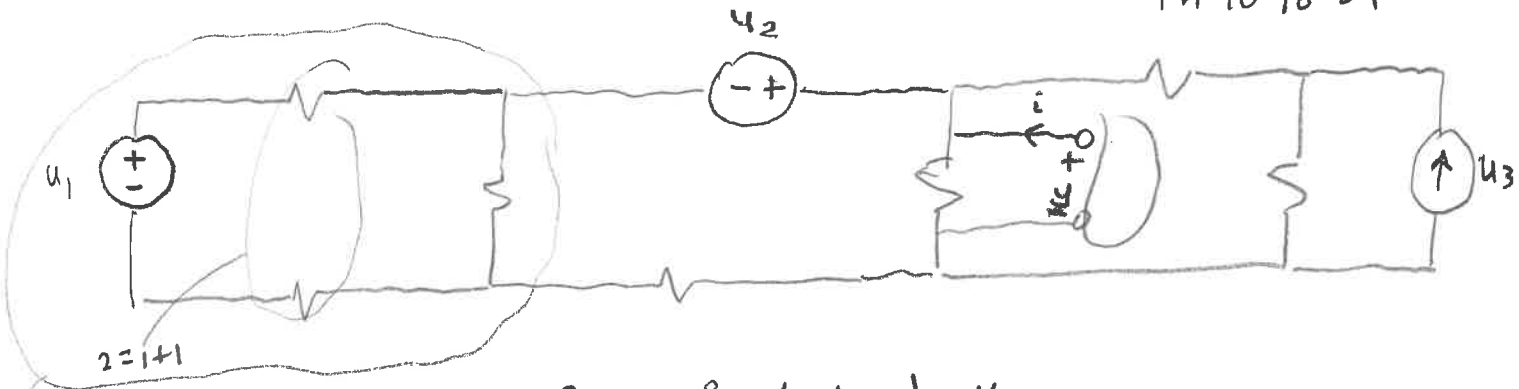
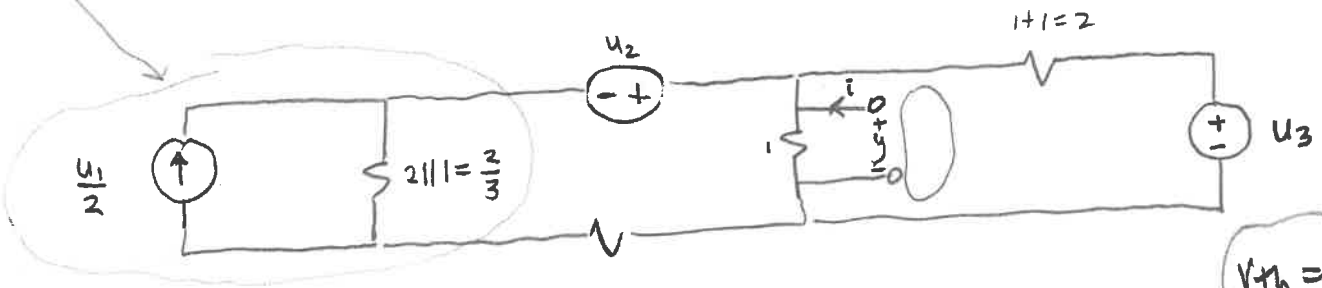
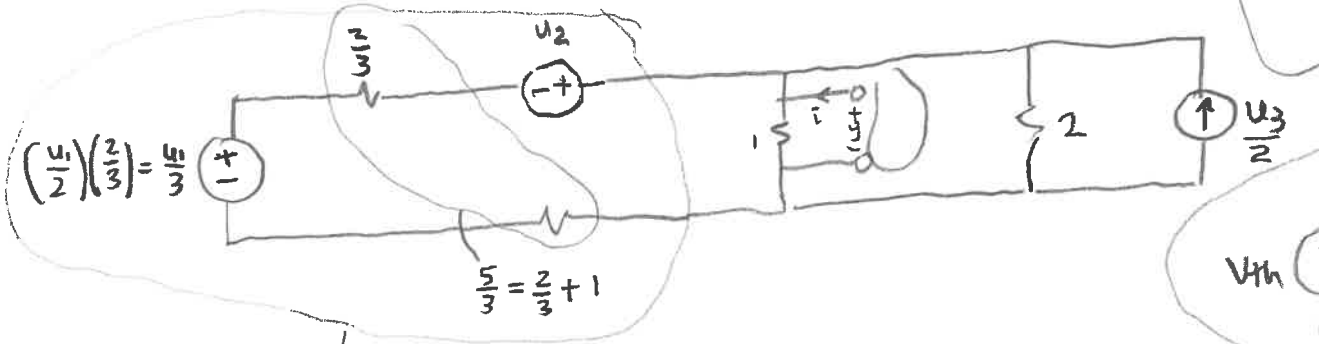


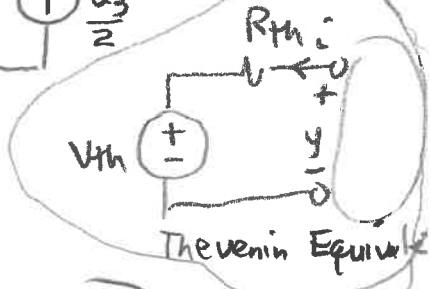

Find a Norton & Thevenin Equivalent at y

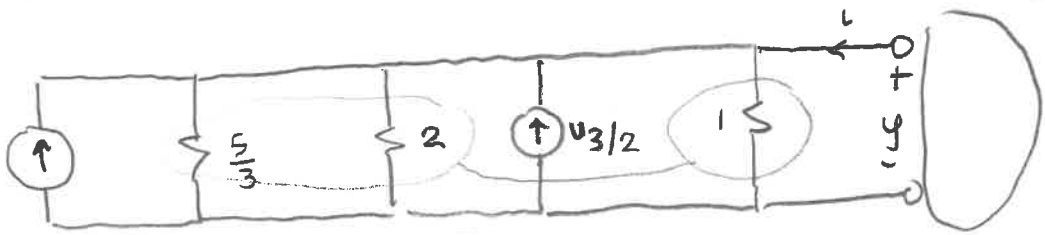
solution via source transformations



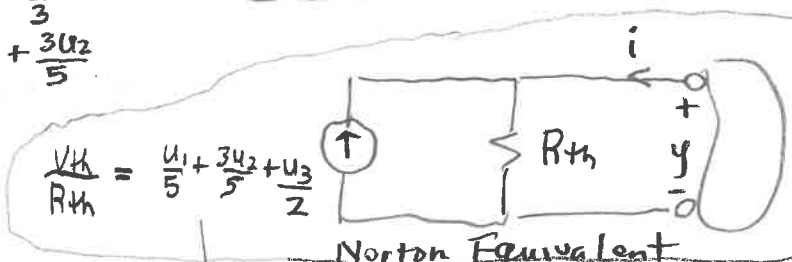

$V_{th} = \left(\frac{2}{21}\right)u_1 + \left(\frac{6}{21}\right)u_2 + \left(\frac{5}{21}\right)u_3$




$\frac{\left(\frac{u_1}{3}+u_2\right)}{\frac{5}{3}} = \frac{u_1}{5}+\frac{3u_2}{5}$




$\frac{V_{th}}{R_{th}} = \frac{u_1}{5}+\frac{3u_2}{5}+\frac{u_3}{2}$

$R_{th} = \left(\frac{5}{3}\right)\|(2)\|(1)$

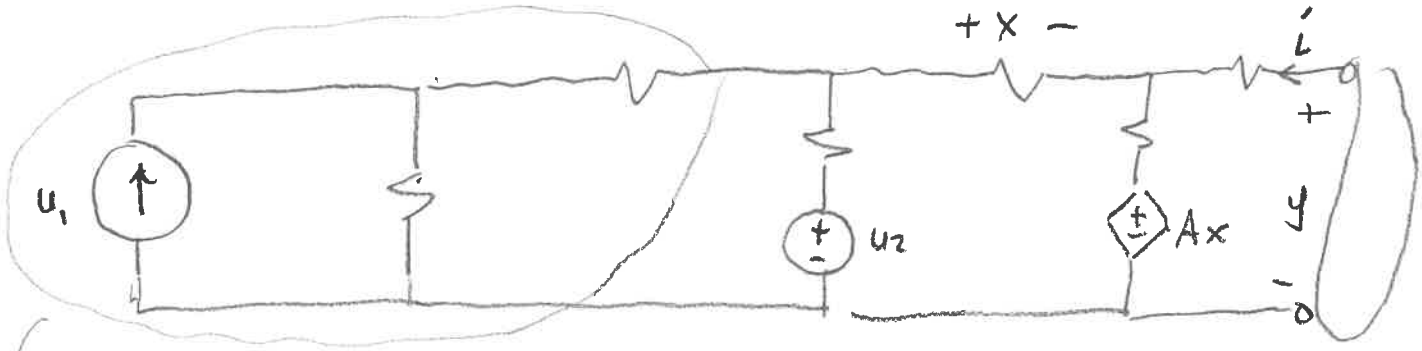
$= \left(\frac{5}{3}\right)\|\left(\frac{2}{3}\right) = \frac{\left(\frac{5}{3}\right)\left(\frac{2}{3}\right)}{\frac{7}{3}}$

$R_{th} = \frac{10}{21}$

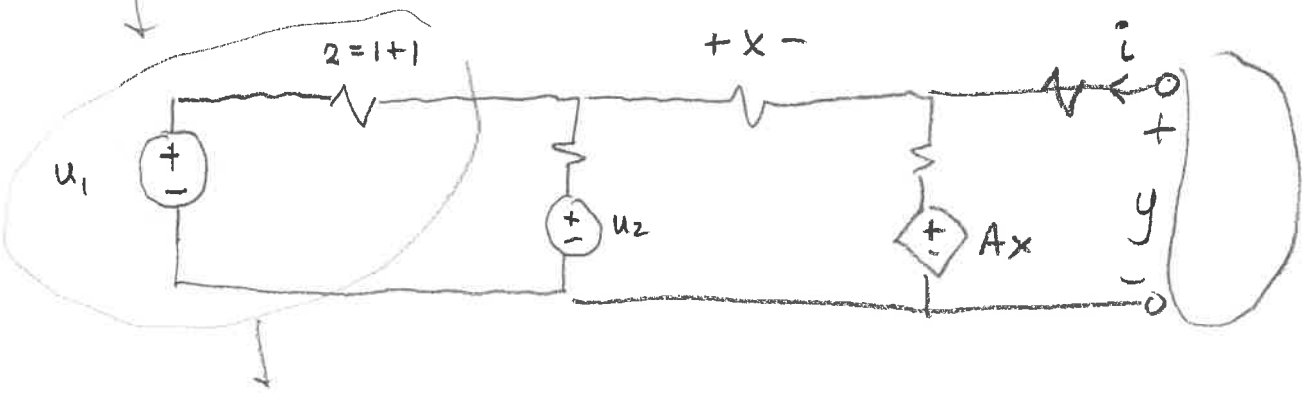
$V_{th} = \frac{10}{21}\left(\frac{u_1}{5}+\frac{3u_2}{5}+\frac{u_3}{2}\right)$

# Problem 5 (Thevenin & Norton Equivalent)

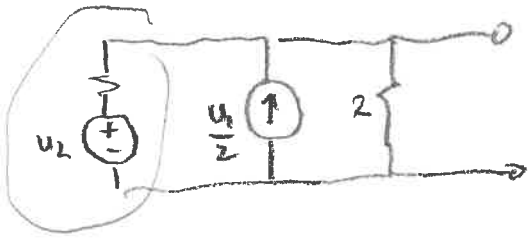
Find Thevenin & Norton equivalent at y (All R's = 1)

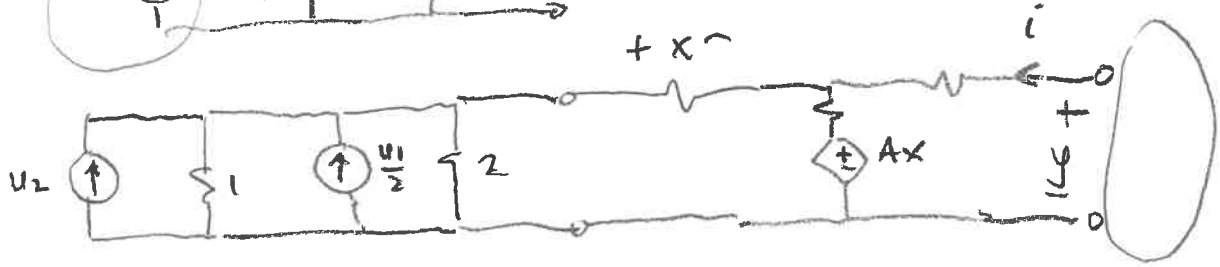



solution using some source transformations







# Problem 5

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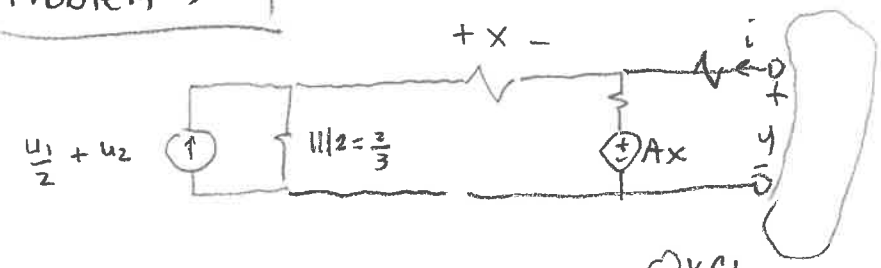
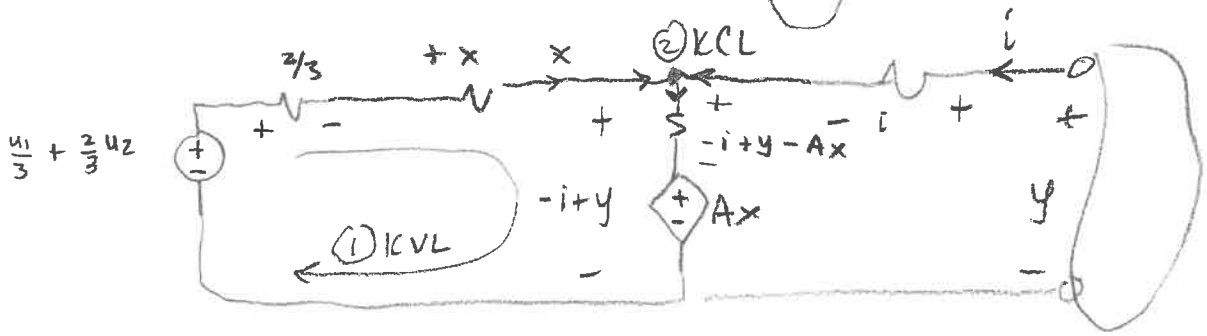



① KVL = $\frac{u_1}{3} + \frac{2u_2}{3} = \frac{2}{3}x + x + (-i+y)$

$\Rightarrow y = i - \frac{5}{3}x + \frac{u_1}{3} + \frac{2u_2}{3}$

$\Rightarrow x = \frac{3}{5}\left[i - y + \frac{u_1}{3} + \frac{2u_2}{3}\right]$

$x = \frac{3}{5}(i-y) + \frac{u_1}{5} + \frac{2u_2}{5}$

② KPL = $x + i = -i + y - Ax$

$\Rightarrow y = 2i + (1+A)x$

$= 2i + (1+A)\left[\frac{3}{5}(i-y) + \frac{u_1}{5} + \frac{2u_2}{5}\right]$

$\Rightarrow y\left[1 + (1+A)\frac{3}{5}\right] = i\left[2 + (1+A)\frac{3}{5}\right] + \frac{(1+A)}{5}[u_1 + 2u_2]$

$\Rightarrow y\left[\underbrace{5+3}_{8} + 3A\right] = i\left[\underbrace{10+3}_{13} + 3A\right] + (1+A)[u_1 + 2u_2]$

$\Rightarrow y = \left[\frac{13+3A}{8+3A}\right] i + \left[\frac{1+A}{8+3A}\right][u_1 + 2u_2]$

$R_{th}$ $V_{th}$

Thev Equiv: $R_{th}$, $V_{th}$, y

Norton Equiv: $\frac{V_{th}}{R_{th}}$, $R_{th}$, y

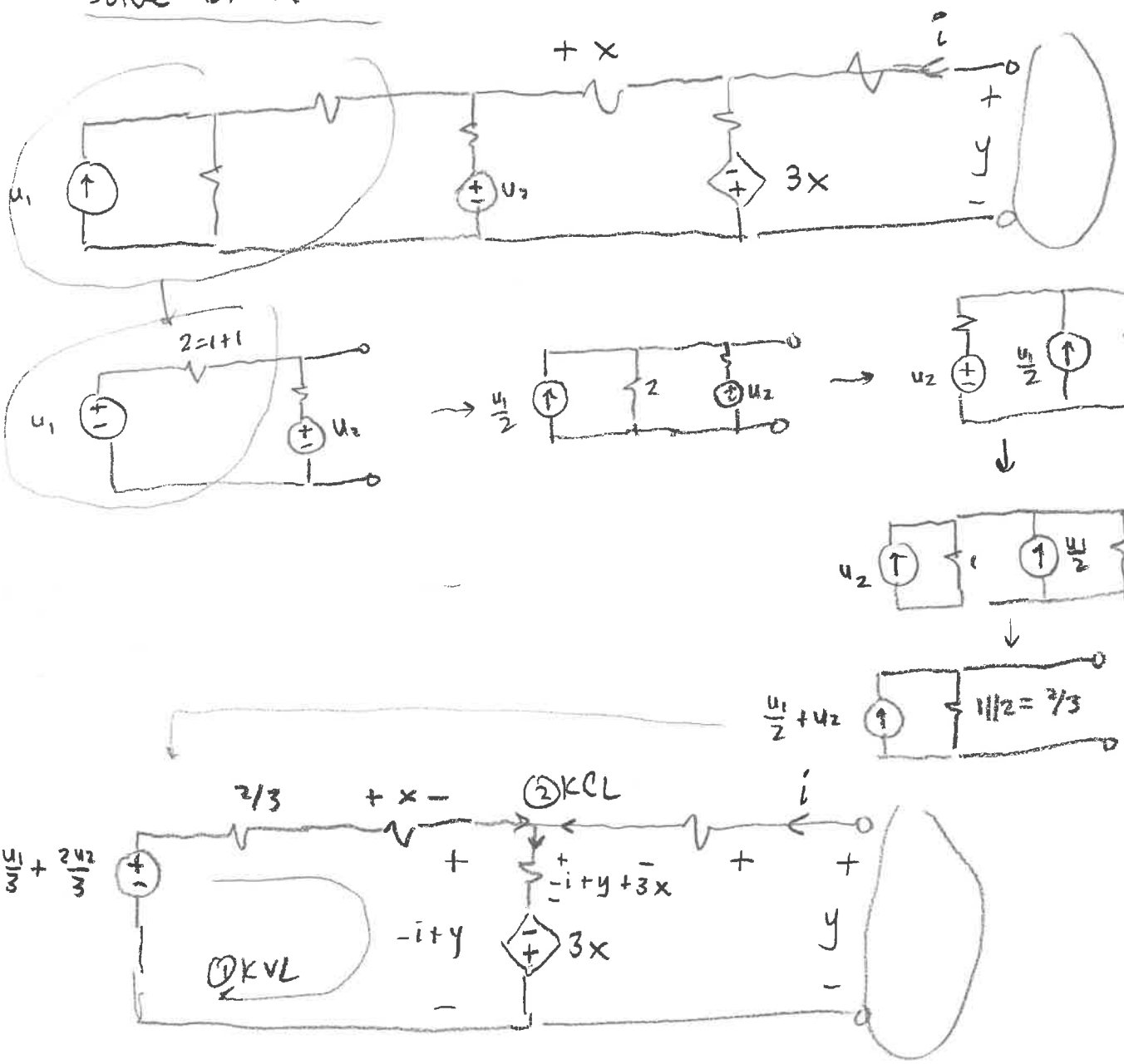
For $A = -3$ $\quad R_{th} = \frac{13-9}{8-9} = \frac{4}{-1} = -4$

$V_{th} = \left(\frac{1-3}{8-9}\right)(u_1 + 2u_2) = \left(\frac{-2}{-1}\right)(u_1 + 2u_2) = 2u_1 + 4u_2$

# Problem 5



Solve for $A = -3$




① KVL = $\boxed{\frac{u_1}{3}+\frac{2u_2}{3} = \frac{2}{3}x + x + (-i+y)}$ $\Rightarrow$ $\frac{5}{3}x = i-y+\frac{u_1}{3}+\frac{2u_2}{3}$

$\Rightarrow x = \frac{3}{5}(i-y)+\frac{u_1}{5}+\frac{2u_2}{5}$

② KCL = $\boxed{\vec{x}+\overleftarrow{i} = -i+y+3x}$

$\Rightarrow y = 2i-2x = 2i-2\left[\frac{3}{5}(i-y)+\frac{u_1}{5}+\frac{2u_2}{5}\right]$

# Problem 5

$$\Rightarrow y\left[1 - 2\left(\tfrac{3}{5}\right)\right] = i\left[2 - 2\left(\tfrac{3}{5}\right)\right] - 2\left[\tfrac{u_1}{5} + \tfrac{2u_2}{5}\right]$$

$$\Rightarrow y\left[5 - 6\right] = i\left[10 - 6\right] - 2\left[u_1 + 2u_2\right]$$

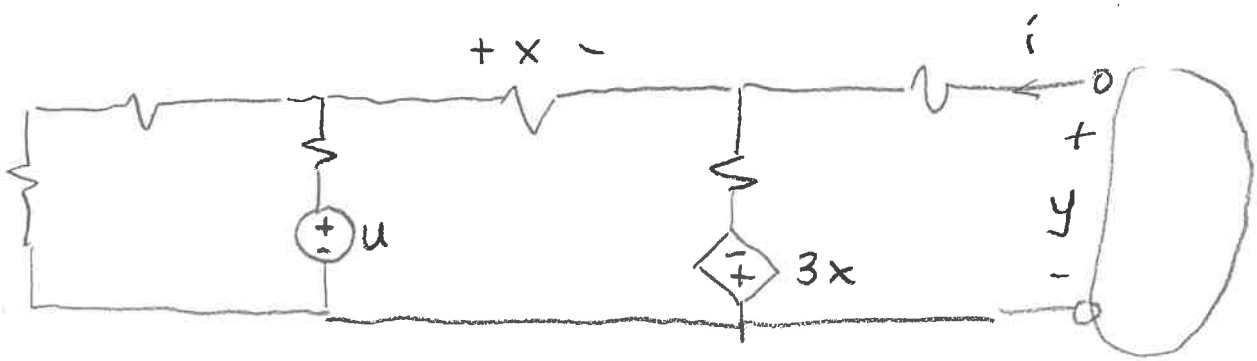
$5 - 6 = -1$

$$\Rightarrow \boxed{y = -4i + 2u_1 + 4u_2}$$

$R_{th} = -4$ $\qquad$ $V_{th} = 2u_1 + 4u_2$

## Final Circuit for Exam 1 (Fall 2024)

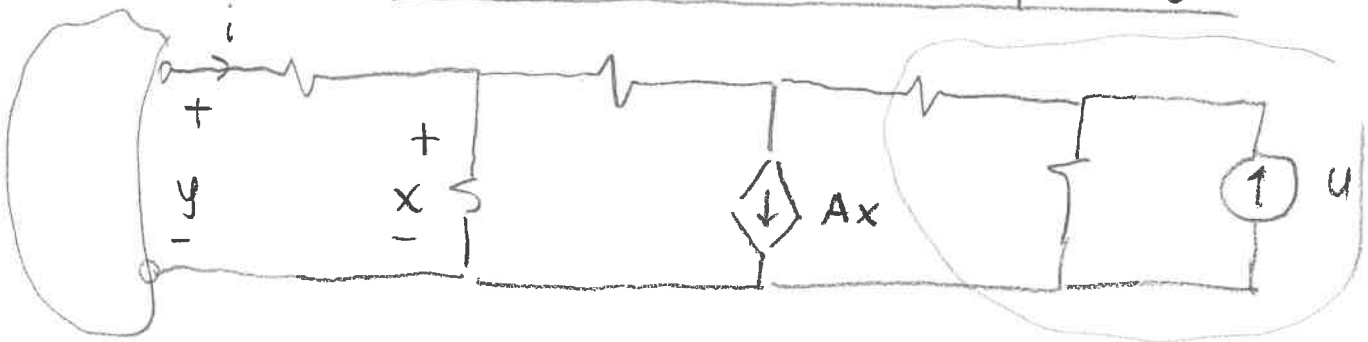
(set $u_1 = 0$)

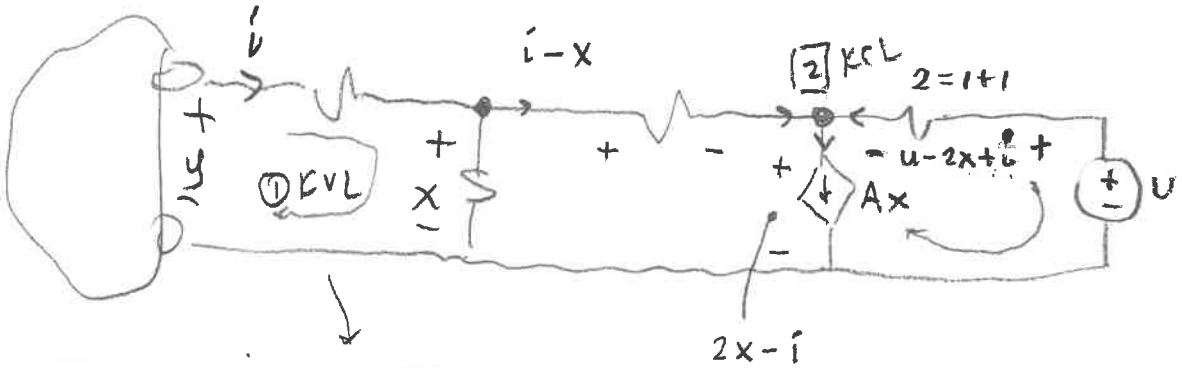



$u_1 = 0$, $u_2 = u$ $\Rightarrow$ $\boxed{y = -4i + 4u}$

# Problem 6 (Thevenin & Norton Equivalent) 10/13

Find Thevenin & Norton Eq at y




1 KVL = $y = i + x$

$x = y - i$

2 KCL = $\rightarrow \downarrow \leftarrow$

$$(i - x) + \left(\frac{u - 2x + i}{2}\right) = Ax$$

$$2i - 2x + u - 2x + i = 2Ax$$

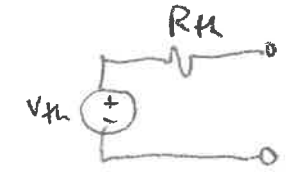
$$3i + u = (4 + 2A)x$$

$$3i + u = (4 + 2A)[y - i]$$

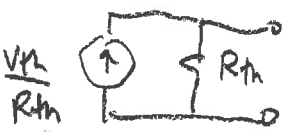
$$(4 + 2A)y = i[3 + 4 + 2A] + u$$

$$y = \left[\frac{7 + 2A}{4 + 2A}\right] i + \left[\frac{1}{4 + 2A}\right] u$$

$R_{th}$ | $V_{th}$




Thevenin Equiv




Norton Equiv

$A = -3$ $\quad R_{th} = \frac{7 - 6}{4 - 6} = -\frac{1}{2}$ $\quad V_{th} = -\frac{1}{2}u$

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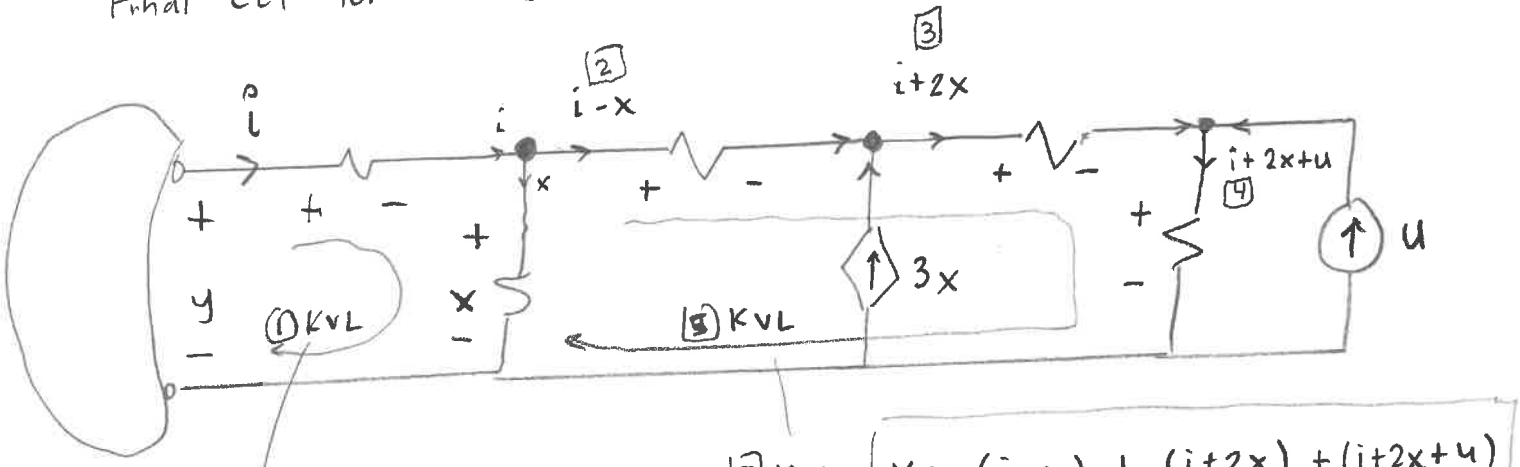
# Problem 6 (Thevenin & Norton Equiv) 11/13

Final Cct for $A = -3$:




1 KVL: $y = i + x$

$\Rightarrow x = y - i$

5 KVL: $x = (i-x) + (i+2x) + (i+2x+u)$

$\Downarrow$

$0 = 3i + u + 2x$

$= 3i + u + 2(y-i)$

$= i + u + 2y$

$\Rightarrow y = -\left(\frac{1}{2}\right) i + \left(-\frac{1}{2}\right) u$

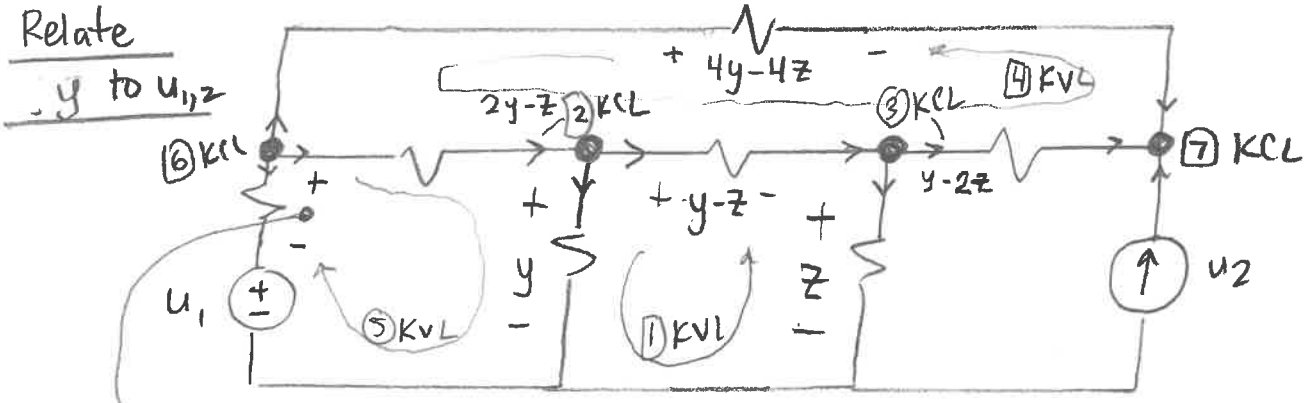
$R_{Th} = -\frac{1}{2}$

$V_{Th} = -\left(\frac{1}{2}\right) u$

# Problem 7 (Need Additional Variable) (All R's = 1)

12/13

Relate
- $y$ to $u_{1,2}$




5 $(2y-z)+(y)-u_1 = 3y-z-u_1$

6 KCL = $(3y-z-u_1) + (2y-z) + (4y-4z) = 0$

$\rightarrow 9y-6z = u_1 \longrightarrow 6z = 9y-u_1$

7 KCL = $(y-2z) + (4y-4z) + u_2 = 0$

$\rightarrow 5y-6z+u_2 = 0 \longrightarrow 6z = 5y+u_2$

$9y-u_1 = 5y+u_2$

$4y = u_1+u_2$

$$y = \frac{1}{4}u_1 + \frac{1}{4}u_2$$

$$y = \left(\frac{6}{24}\right)u_1 + \left(\frac{6}{24}\right)u_2$$

$6z = 9y-u_1 = \frac{9}{4}u_1 + \frac{9}{4}u_2 - u_1$

$= \frac{5}{4}u_1 + \frac{9}{4}u_2$

$$z = \left(\frac{5}{24}\right)u_1 + \left(\frac{9}{24}\right)u_2$$

$y-z = \left(\frac{6}{24}\right)u_1 + \left(\frac{6}{24}\right)u_2 - \left(\frac{5}{24}\right)u_1 - \left(\frac{9}{24}\right)u_2$

$= \left(\frac{1}{24}\right)u_1 - \frac{3}{24}u_2$

$4(y-z) = \left(\frac{4}{24}\right)u_1 - \left(\frac{12}{24}\right)u_2$

# Problem 7

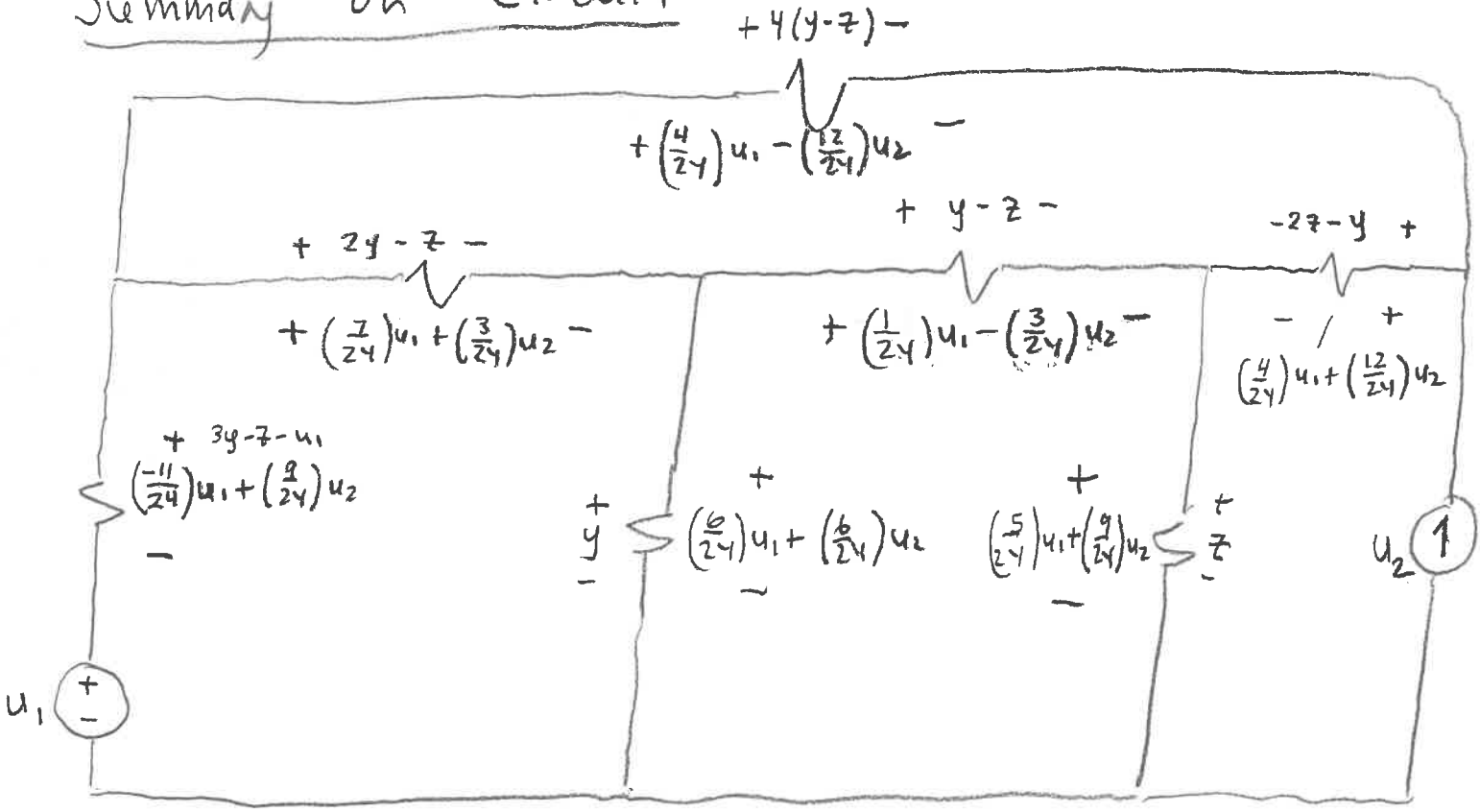
$$y - 2z = \left(\frac{6}{24}\right)u_1 + \left(\frac{6}{24}\right)u_2 - \left(\frac{10}{24}\right)u_1 - \left(\frac{18}{24}\right)u_2 = \left(\frac{-4}{24}\right)u_1 + \left(\frac{-12}{24}\right)u_2$$

$$2y - z = \left(\frac{12}{24}\right)u_1 + \left(\frac{12}{24}\right)u_2 - \left(\frac{5}{24}\right)u_1 - \left(\frac{9}{24}\right)u_2 = \left(\frac{7}{24}\right)u_1 + \left(\frac{3}{24}\right)u_2$$

$$3y - z - u_1 = \left(\frac{18}{24}\right)u_1 + \left(\frac{18}{24}\right)u_2 - \left(\frac{5}{24}\right)u_1 - \left(\frac{9}{24}\right)u_2 = \left(\frac{13}{24}\right)u_1 + \left(\frac{9}{24}\right)u_2 - \frac{24}{24}u_1$$

$$-u_1 = \left(\frac{-11}{24}\right)u_1 + \left(\frac{9}{24}\right)u_2$$

Summary on Circuit:




Note: KVL & KCL are satisfied everywhere! (& Ohm)

... consistent summary on the circuit suggests that our solution for y & z are correct!!!