EEE 202 Fall 2024 Final Exam Fri 12/13/24
with SOLUTIONS!

EXAM RULES

1) One 8.5" x 11" sheet permitted;
   otherwise closed books, closed notes, open minds!

2) Scientific calculator is permitted; no computers and no phone!

3) **NO PHONES !!!**
   Phones should not be visible at all!
   A visible phone will result in a zero grade for the exam!

4) Write FULL NAME legibly on each page provided.

5) **PLEASE SHOW ALL WORK !!!**
   This is essential to receive partial credit!

6) Please do not submit multiple answers. You must pick an answer!!

7) Write your solutions on the sheets provided.
   No other paper/sheets/pages should be used!

8) Clearly label voltages and currents on the circuits provided.

9) Use the variables provided!
   No additional variables should be used!

10) Unreadable work will receive NO CREDIT.

11) Please place important equations and answers within boxes as we have done in lecture.

12) Please be careful with your algebra, signs, etc.

13) Please turn in your solutions to me at the end of the period!

14) **PLEASE DO NOT CHEAT !!!**

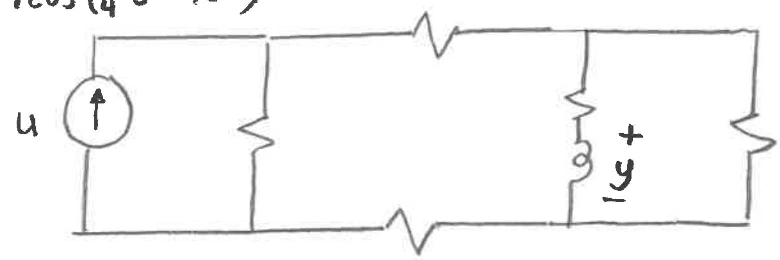
From the desk of Dr. A.A. Rodriguez

Problem # 1

Determine H, diff eq, t_s, y_{ss}, y ; Plot $|H(j\omega)|$ & $\angle H(j\omega)$

(All R, L = 1)

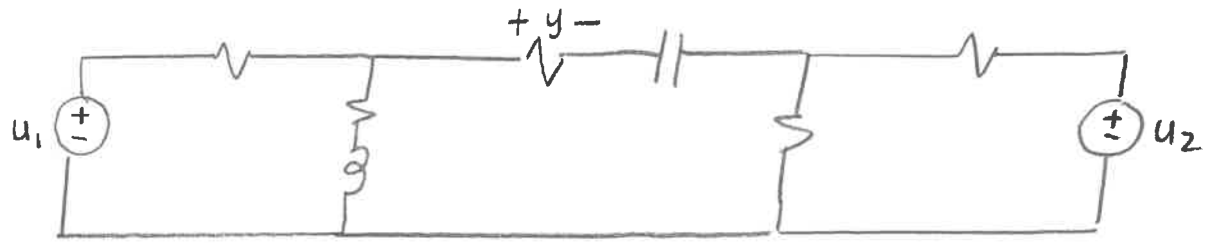
$u = -3 + 7\cos\left(\frac{7}{4}t - 45^\circ\right)$



$\boxed{\text{Problem 1}}$

Problem 2

Determine poles, ts, $H_i(0)$, $H_i(\infty)$ $i=1,2$ (All $R, L, C = 1$)



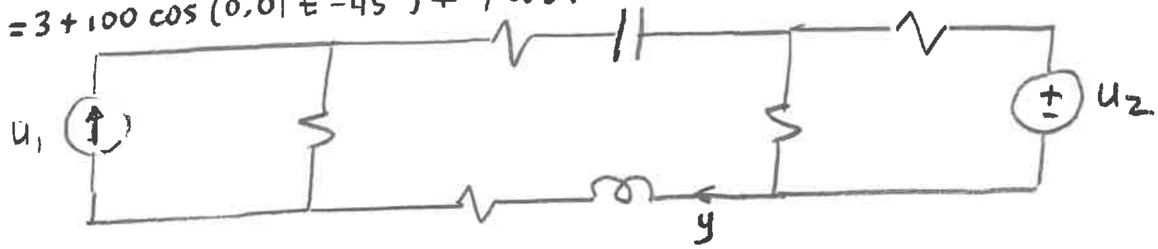
Problem 2

Problem 3

Determine $H_{1,2}$, diff eq, t_s, y_{ss} (All $R, L, C = 1$)

$u_1 = 3 + 100\cos(0.01t - 45°) + 7\cos(t + 20°)$

$u_2 = -4 + 2\sin(100t + 45°)$



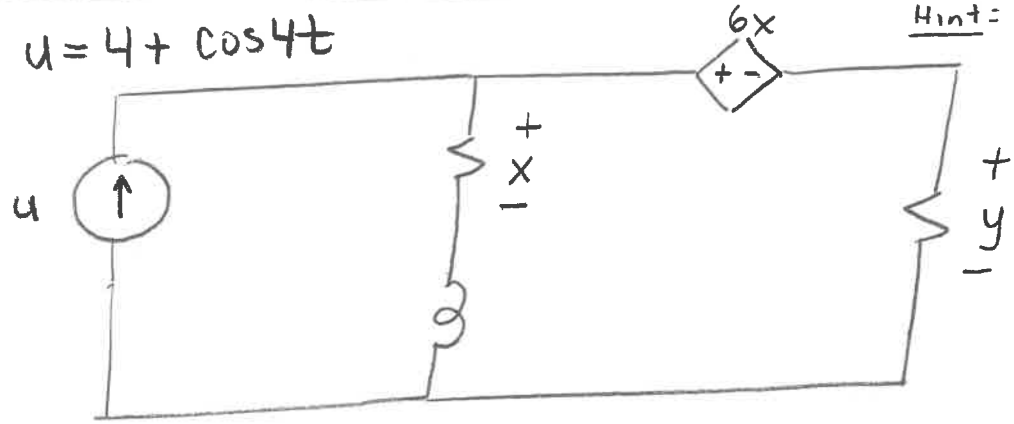
Problem 3

Problem 4

Determine H, diff eq, y, y_{ss} (All $R, L = 1$)

Hint: Use KVL, KCL, Ohm!

$u = 4 + \cos 4t$



Problem 4

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Problem 5

Determine t_s, y_{ss}, y

Note:
Show how to calculate all coefficients associated with constants, sinusoids, & instabilities in y.
You are expected to do the easy calculations!

$$H(s) = \frac{s^2+1}{(s+2)(s^2-6s+25)(s^2+2s+4)}$$

$$u(t) = 5 - 60\sin(t+30) + 7\cos(100t+45°)$$

Problem 5

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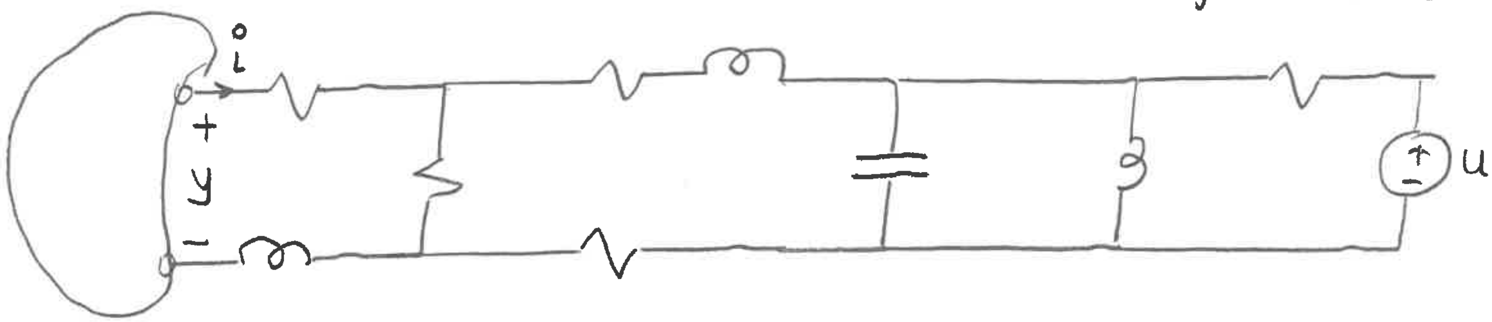
Problem 6

(All R, L, C = 1)

Determine an s-domain Thevenin equivalent at y

Must specify Z_{th} & V_{th} ... emphasize process ... NOT algebra!!!



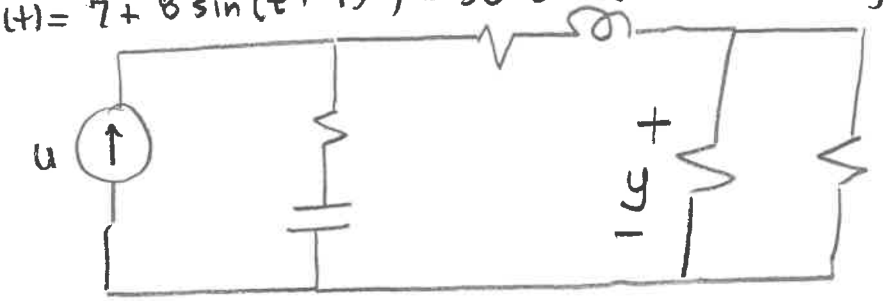
Problem 6

Problem 7

Determine H, y_{ss}, t_s

Note: (All $R, L, C = 1$)
Determine all coefficients in y_{ss} (approximately)

$u(t) = 7 + 8\sin(t + 45°) - 50\cos(100t - 80°)$

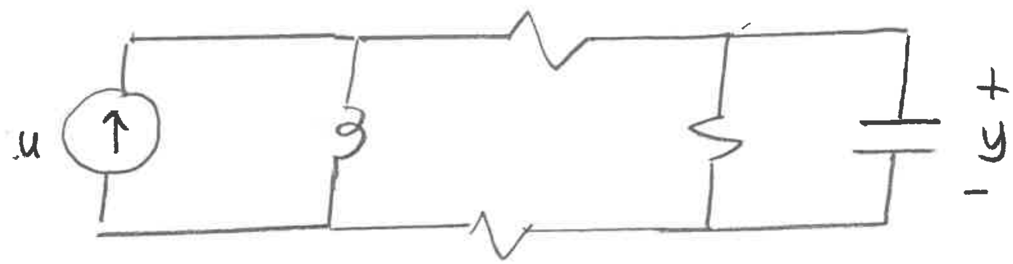


Problem 7

Problem 8

Determine & plot $|H(j\omega)|$ & $\angle H(j\omega)$

(All $R, L, C = 1$)



Problem 8

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Problem 1

Determine H, diff eq, t_s, y_{ss}, y; Plot $|H(j\omega)|$, $\angle H(j\omega)$

$u = -3 + 7\cos(\frac{7}{4}t - 45^\circ)$

(All $R, L = 1$)

$M \leftrightarrow M/s$

$M \sin\omega_0 t \leftrightarrow \frac{M\omega_0}{s^2+\omega_0^2}$

$M\cos\omega_0 t \leftrightarrow \frac{Ms}{s^2+\omega_0^2}$

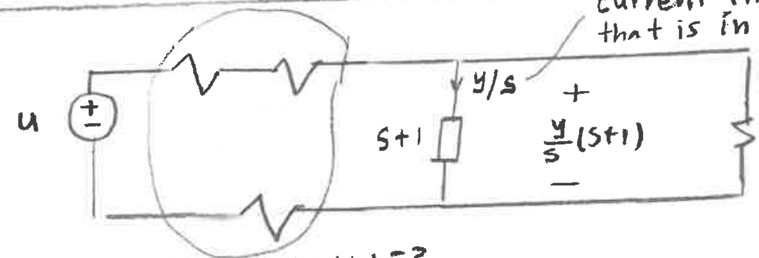
solution via source transformations

current through inductor & 1Ω resistor that is in series with it

$\cos(x-y) = \cos x\cos y + \sin x \sin y$

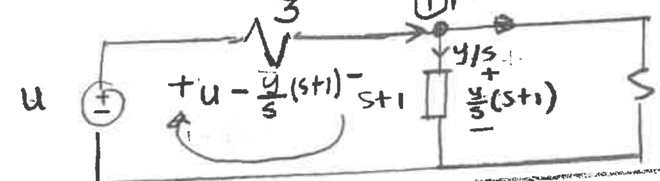
Note:

$u(t) = -3 + 7(\cos\frac{7}{4}t)(\cos 45) + 7(\sin\frac{7}{4}t)(\sin 45)$, $\frac{1}{\sqrt{2}}$



$U(s) = -\frac{3}{s} + 7\left(\frac{s}{s^2+(\frac{7}{4})^2}\right)\frac{1}{\sqrt{2}} + 7\left(\frac{7/4}{s^2+(\frac{7}{4})^2}\right)\frac{1}{\sqrt{2}}$

$= \frac{\text{num}}{s(s^2+(\frac{7}{4})^2)}$



1 KCL =

$$\boxed{\left(\frac{u - \frac{y}{s}(s+1)}{3}\right) = \left(\frac{y}{s}\right) + \left(\frac{y}{s}(s+1)\right)}$$

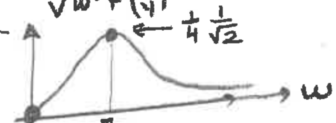
$y\left[\frac{1}{3}\left(\frac{s+1}{s}\right) + \frac{1}{s} + \frac{s+1}{s}\right] = \frac{1}{3}u$

$y\left[s+1 + 3 + 3s+3\right] = su$

$4s+7 = 4(s+\frac{7}{4})$

$H(j\omega) = \frac{\frac{1}{4}j\omega}{j\omega + \frac{7}{4}}$

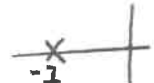
$|H(j\omega)| = \frac{\frac{1}{4}\omega}{\sqrt{\omega^2+(\frac{7}{4})^2}}$



$$\boxed{H(s) = \frac{y}{u} = \frac{\frac{1}{4}s}{s+7/4}}$$

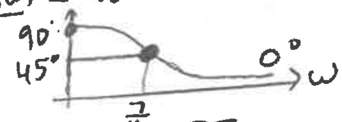
$\Rightarrow \Phi(s) = s + \frac{7}{4} = 0$

$\Rightarrow$ pole at $s = -\frac{7}{4}$



stable

$\angle H(j\omega) = 90^\circ - \tan^{-1}\left(\frac{\omega}{7/4}\right)$



MOTF

$\Rightarrow \dot{y} + \frac{7}{4}y = \frac{1}{4}\dot{u}$

$\Rightarrow t_s = \frac{5}{|\text{pole}|} = \frac{5}{\frac{7}{4}} = \frac{20}{7}$ sec

$$\boxed{y_{ss} = -3H(0) + 7|H(j\tfrac{7}{4})|\cos\left(\tfrac{7}{4}t - 45^\circ + \angle H(j\tfrac{7}{4})\right)}$$

$H(0) = 0$ $\quad H(j\frac{7}{4}) = \frac{1}{4}\left[\frac{j\frac{7}{4}}{j\frac{7}{4}+\frac{7}{4}}\right] = \frac{1}{4}\left[\frac{1e^{j90}}{\sqrt{2}e^{j45}}\right] = \frac{1}{4\sqrt{2}}e^{j(90-45^\circ)}$

$t_s = \frac{5}{\frac{7}{4}} = \frac{20}{7}$ sec

$Y = HU = \left[\frac{\frac{1}{4}s}{s+\frac{7}{4}}\right]\left[\frac{\text{num}}{s(s^2+(\frac{7}{4})^2)}\right] = \frac{A}{s} + \frac{B}{s+\frac{7}{4}} + \left[\frac{C}{s-j\frac{7}{4}} + *\right]$

$$\boxed{y = A + Be^{-\frac{7}{4}t} + 2|C|\cos(\tfrac{7}{4}t + \angle C)}$$

y_{ss} (MOTF)

$A = \lim_{s\to 0} sY(s) = sH(s)\left[-\frac{3}{s} + \frac{\cdot}{s^2+(\frac{7}{4})^2}\right]\Big|_{s=0} = -3H(0)$ MOTF

$B = \lim_{s\to \frac{7}{4}}(s+\frac{7}{4})Y(s)$

$C = \lim_{s\to j\frac{7}{4}}(s-j\frac{7}{4})Y(s) = \frac{7|H(j\frac{7}{4})|}{2}e^{j(-45^\circ + \angle H(j\frac{7}{4}))}$

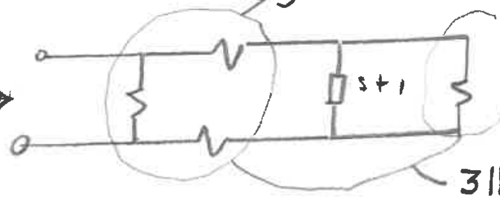
MOTF

Note: You need to compute num in $U(s)$ to actually show that the right hand side holds!

Problem 1

series: $1+1+1 = 3$

pole check:
set $u=0$



$\Rightarrow$

$3 \| 1 = \frac{3}{4}$

$Z = s + 1 + 3/4$
$= s + 7/4 = 0$

$\Rightarrow$ pole $= -7/4$

Good to always CHECK!

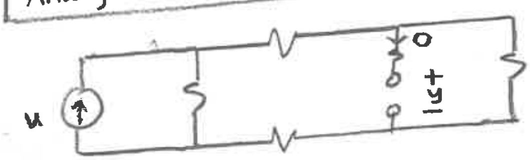
Gain Checks:

$\boxed{\text{Analysis at } s=0}$ $Z_L = sL = 0$ (short)

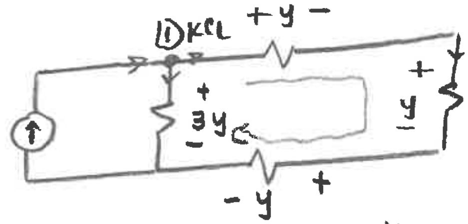
$y = 0 = 0\,u \Rightarrow \boxed{H(0) = 0}$

Can't get EASIER than this!

$\boxed{\text{Analysis at } s=\infty}$ $Z_L = sL = \infty$ (open)



$\Rightarrow$



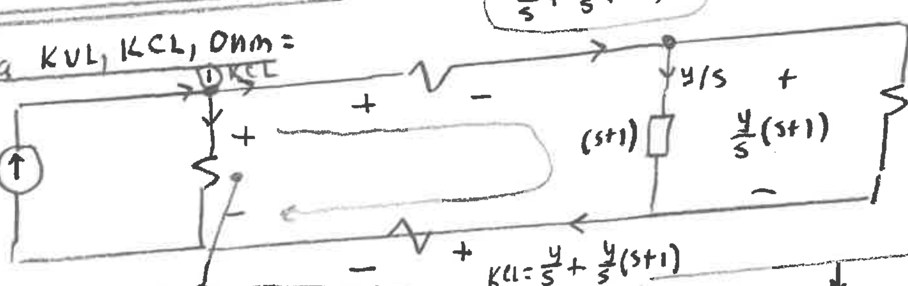
① KCL: $u = 3y + y = 4y \Rightarrow y = \frac{1}{4}u$

$\Rightarrow \boxed{H(\infty) = \frac{1}{4}}$

$\Rightarrow$ Good to always check gains at dc ($s=0$) & $s=\infty$!

solution via KVL, KCL, Ohm:

KCL: $\frac{y}{s} + \frac{y}{s}(s+1)$



KCL: $\frac{y}{s} + \frac{y}{s}(s+1)$

$\left[\frac{y}{s} + \frac{y}{s}(s+1)\right] + \left[\frac{y}{s}(s+1)\right] + \left[\frac{y}{s} + \frac{y}{s}(s+1)\right]$

$= \frac{2y}{s} + \frac{3y}{s}(s+1)$

① KCL: $\boxed{u = \left[\frac{2y}{s} + \frac{3y}{s}(s+1)\right] + \left[\frac{y}{s} + \frac{y}{s}(s+1)\right]}$

$\Rightarrow y\,[2 + 3s + 3 + 1 + s + 1] = s\,u$

$4s + 7$

$\Rightarrow \boxed{H(s) = \frac{\frac{1}{4}s\,u}{s + \frac{7}{4}}}$

Just like what we got before!

$\boxed{2 \| 13}$

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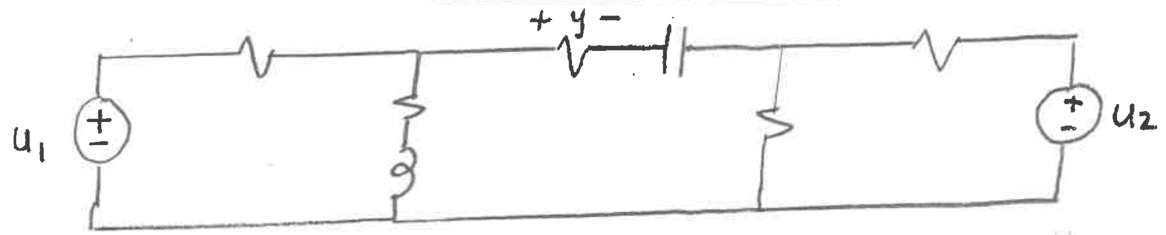
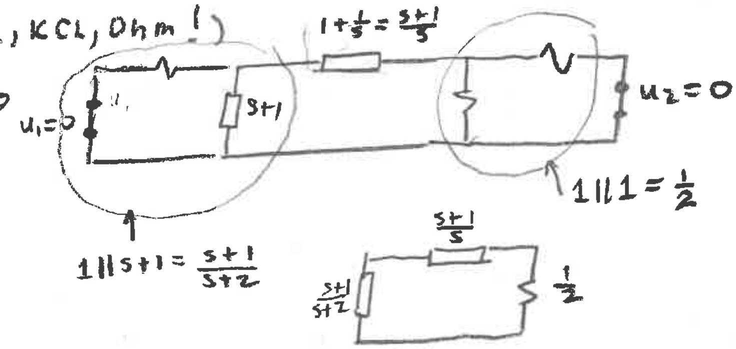
Problem 2

Determine poles, t_s, $H_i(0)$, $H_i(\infty)$ $i=1,2$ (All $R, L, C = 1$)



solution for poles, ts (without KVL, KCL, Ohm!)

Set $u_1 = 0$ (short), $u_2 = 0$ (short) $\Rightarrow$



$$Z = \frac{s+1}{s+2} + \frac{s+1}{s} + \frac{1}{2} = \frac{s(s+1) + (s+1)(s+2) + \frac{1}{2}s(s+2)}{s(s+2)} = \frac{s^2+s+s^2+3s+2+\frac{1}{2}s^2+s}{s(s+2)}$$

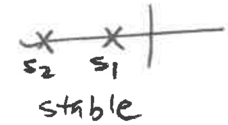
$$\frac{5}{2}s^2 + 5s + 2 = \frac{5}{2}\left[s^2+2s+\frac{4}{5}\right]$$

$$\Rightarrow \Phi = s^2 + 2s + \frac{4}{5} = 0$$

poles $\Rightarrow$ $s_{1,2} = \dfrac{-2 \pm \sqrt{4 - \frac{16}{5}}}{2}$

$$= -1 \pm \sqrt{\left(1-\frac{4}{5}\right)}\ \frac{1}{5}$$

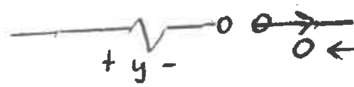
$$= -1 \pm \frac{1}{\sqrt{5}}$$



$$t_s = \frac{5}{|s_1|} = \frac{5}{1-\frac{1}{\sqrt{5}}}$$

Analysis at $s=0$

$Z_L = sL = 0$ (short)
$Z_C = \frac{1}{sC} = \infty$ (open)



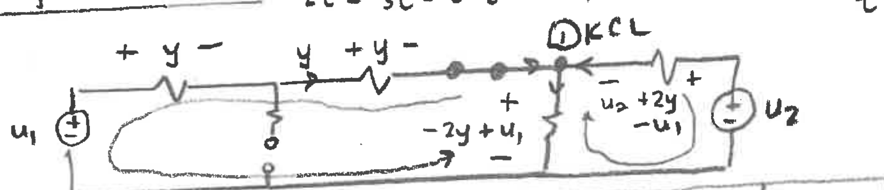
cap is OPEN!

$\Rightarrow$ $y = 0 = 0u_1 + 0u_2$

$\Rightarrow$ $\boxed{H_1(0) = H_2(0) = 0}$

Analysis at $s=\infty$

$Z_L = sL = \infty$ (open)
$Z_C = \frac{1}{sC} = 0$ (short)



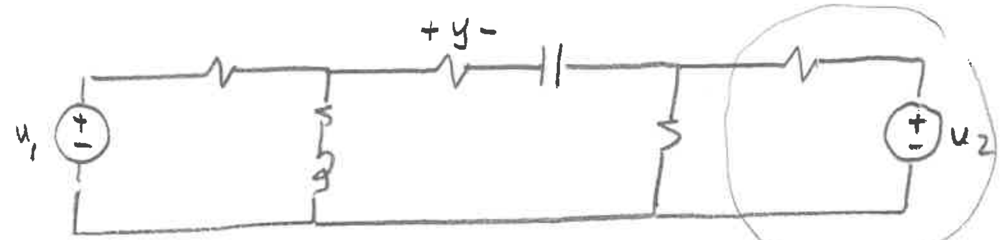
① KCL: $\boxed{y + (u_2 + 2y - u_1) = (-2y + u_1)}$

$\Rightarrow$ $y[\underbrace{1+2+2}_{5}] = u_1[\underbrace{1+1}_{2}] - u_2$

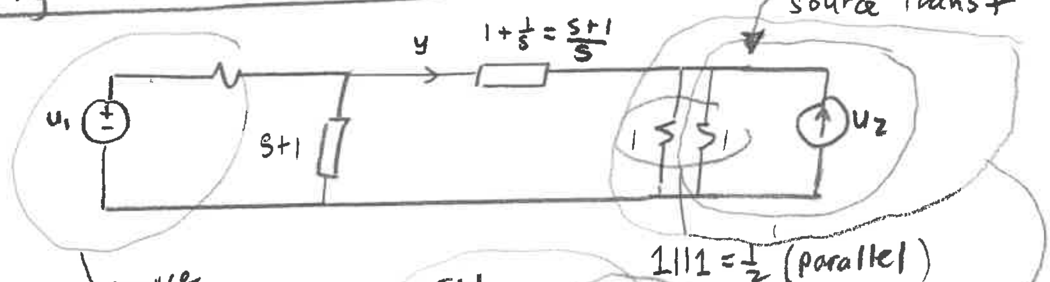
$\Rightarrow$ $y = \left(\frac{2}{5}\right)u_1 + \left(\frac{-1}{5}\right)u_2$ $\Longrightarrow$ $\boxed{H_1(\infty) = \frac{2}{5} \quad H_2(\infty) = \frac{-1}{5}}$

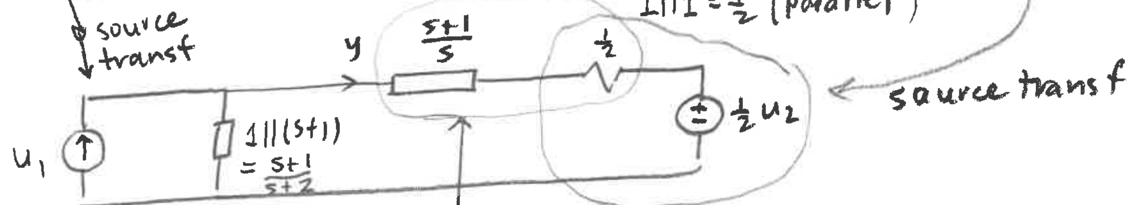
Problem 2

Determine poles, ts, $H_i(0)$, $H_i(\infty)$ $i=1,2$ (All $R, L, C = 1$)



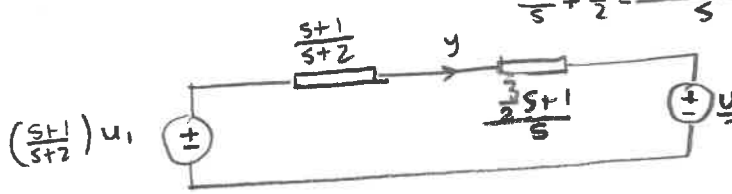
Long Solution: Find $H_{1,2}$!





series

$$\frac{s+1}{s} + \frac{1}{2} = \frac{s+1+\frac{1}{2}s}{s} = \frac{\frac{3}{2}s+1}{s}$$



Ohm $\Rightarrow$

$$y = \frac{\left(\frac{s+1}{s+2}\right)u_1 - \frac{u_2}{2}}{\frac{s+1}{s+2} + \frac{\frac{3}{2}s+1}{s}} = \frac{s(s+1)u_1 - \frac{1}{2}s(s+2)u_2}{s(s+1) + \left(\frac{3}{2}s+1\right)(s+2)}$$

$$s^2 + s + \frac{3}{2}s^2 + 3s + s + 2 = \frac{5}{2}s^2 + 5s + 2 = \frac{5}{2}\left[s^2 + 2s + \frac{4}{5}\right]$$

$$\Rightarrow \boxed{y = \underbrace{\left[\frac{\frac{2}{5}s(s+1)}{s^2 + 2s + \frac{4}{5}}\right]}_{H_1} u_1 + \underbrace{\left[\frac{-\frac{1}{5}s(s+2)}{s^2 + 2s + \frac{4}{5}}\right]}_{H_2} u_2}$$

$\Rightarrow \Phi = s^2 + 2s + \frac{4}{5}$ --- just like before!

$\Rightarrow H_1(0) = H_2(0) = 0$ --- " " "

$\Rightarrow H_1(\infty) = \frac{2}{5}$ $\quad H_2(\infty) = -\frac{1}{5}$ --- " " "

$\Rightarrow$ The previous method used is a great way to get poles, ts, & $H_i(0)$, $H_i(\infty)$ quickly!!!

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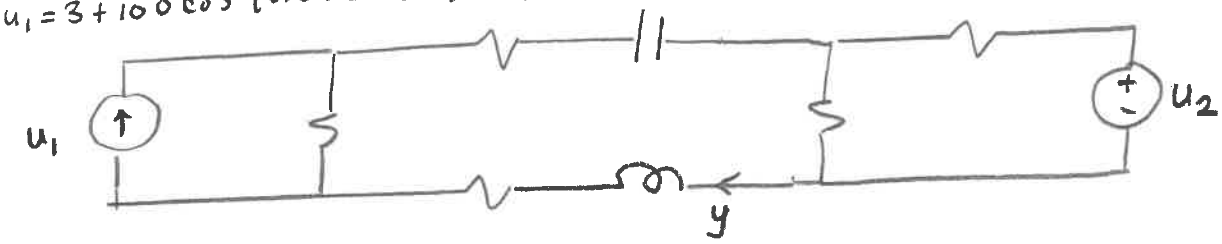
Problem 3

Determine $H_{1,2}$, diff eq, t_s, y_{ss}

$u_1 = 3 + 100\cos(0.01t - 45°) + 7\cos(t + 2°)$ $u_2 = -4 + 2\sin(100t + 45°)$



solution:

Since circuit consists of R, L, Cs & independent sources ($u_{1,2}$), it MUST BE STABLE; i.e. all poles in left half s-plane!

(we'll show this below)

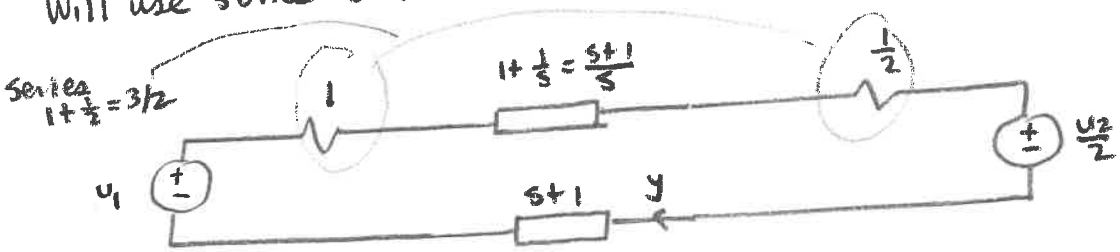
$$\Rightarrow \quad y_{ss} = y_{1ss} + y_{2ss} \quad y_{1ss} \overset{\text{MOTF}}{=} 3H_1(0) + 100|H_1(j0.01)|\cos(0.01t - 45° + \angle H_1(j0.01)) + 7|H_1(j1)|\cos(t + 2° + \angle H_1(j1))$$

$$y_{2ss} \overset{\text{MOTF}}{=} -4H_2(0) + 2|H_2(j100)|\sin(100t + 45° + \angle H_2(j100))$$

... we'll compute the required magnitudes & angles below ---

Now for the circuit analysis -

Will use some source transformations -



$$y \overset{\text{ohm}}{=} \frac{u_1 - \frac{u_2}{2}}{\frac{3}{2} + \left(\frac{s+1}{s}\right) + (s+1)} = \frac{s\left(u_1 - \frac{u_2}{2}\right)}{\underbrace{\frac{3}{2}s + s + 1 + s^2 + s}_{s^2 + \frac{7}{2}s + 1}}$$

because capacitor is open at dC

$\Rightarrow$ $H_1(0) = H_2(0) = 0$

$$y = \underbrace{\left[\frac{s}{s^2 + \frac{7}{2}s + 1}\right]}_{H_1} u_1 + \underbrace{\left[\frac{-\frac{1}{2}s}{s^2 + \frac{7}{2}s + 1}\right]}_{H_2} u_2$$

poles $\Phi = s^2 + \frac{7}{2}s + 1 = 0 \Longrightarrow$

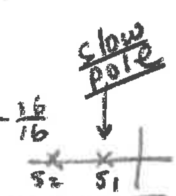
$$s_{1,2} = \frac{-\frac{7}{2} \pm \sqrt{\frac{49}{4} - 4}}{2} = -\frac{7}{4} \pm \sqrt{\frac{49}{16} - 1\cdot\frac{16}{16}} = -\frac{7}{4} \pm \frac{\sqrt{33}}{4}$$

$$\ddot{y} + \frac{7}{2}\dot{y} + y = \dot{u}_1 - \frac{1}{2}\dot{u}_2$$

$H_1(j0.01) \cong j0.01 = 0.1e^{j90°}$ $\quad H_1(j1) = \frac{j1}{\cancel{-1} + \frac{7}{2}j1 + \cancel{1}} = \frac{2}{7}e^{j0°}$

$H_2(j100) \cong \frac{-1/2}{j100} = \frac{\frac{1}{2}e^{j180}}{100e^{j90°}} = \frac{1}{200}e^{j(180-90°)}$

$$\Rightarrow t_s = \frac{5}{|s_1|} = \frac{5}{\frac{7}{4} - \frac{\sqrt{33}}{4}}$$



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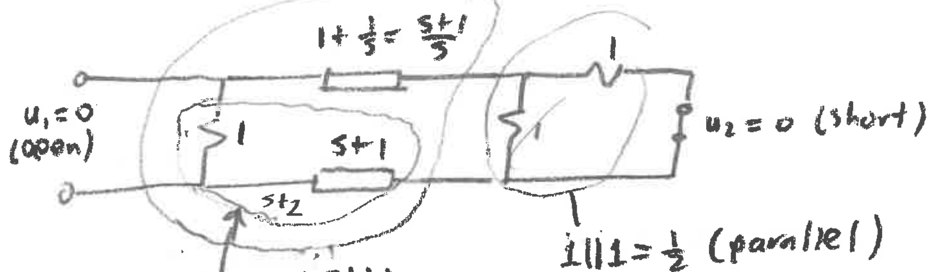
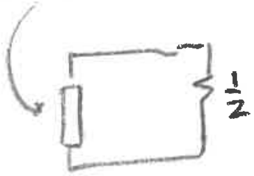
Problem 3

pole check:

Set $u_1 = 0$ (open)
$u_2 = 0$ (short)



series: $(s+2) + \left(\frac{s+1}{s}\right)$



$$Z = (s+2) + \left(\frac{s+1}{s}\right) + \frac{1}{2}$$

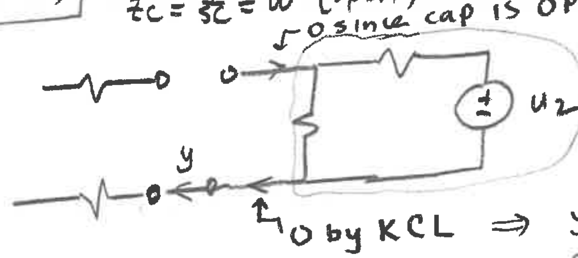
$$= \frac{s^2 + 2s + s + 1 + \frac{1}{2}s}{s} \qquad s^2 + \frac{7}{2}s + 1$$

$\Rightarrow \Phi = s^2 + \frac{7}{2}s + 1$ ... just like before!

Gain Checks:

Analysis at dc $(s=0)$

$Z_L = sL = 0$ (short)
$Z_C = \frac{1}{sC} = \infty$ (open)



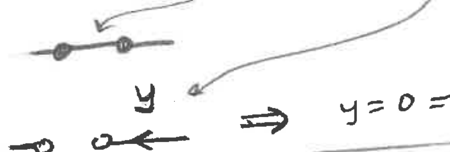
$\Rightarrow y = 0 = 0u_1 + 0u_2$

$\Rightarrow \boxed{H_1(0) = H_2(0) = 0}$

--- just like before!

Analysis at $s = \infty$

$Z_L = sL = \infty$ (open)
$Z_C = \frac{1}{sC} = 0$ (short)



$\Rightarrow y = 0 = 0u_1 + 0u_2$

$\Rightarrow \boxed{H_1(\infty) = H_2(\infty) = 0}$

$\Rightarrow$ Yes ... our H_1 & H_2 satisfy these!

$\Rightarrow$ Based on the above 3 ☺'s, it is reasonable to believe that we computed H_1 & H_2 correctly?

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$\Longrightarrow$ Should try to always do these pole & gain checks!!

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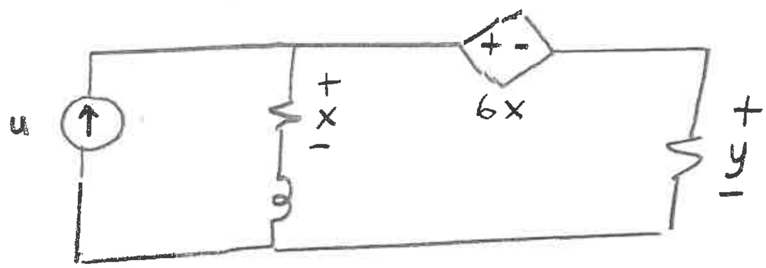
$\boxed{\text{Problem 4}}$

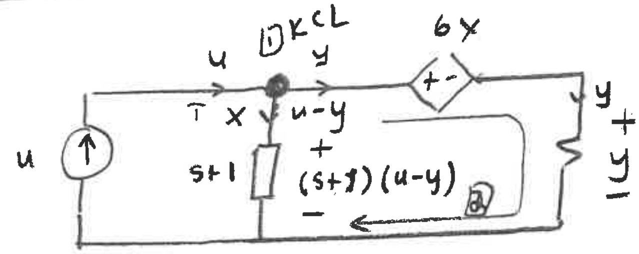
Determine H, diff eq, y, y_{ss} (All $R, L = 1$)

Hint: Use KVL, KCL, Ohm!

$u = 4 + \cos 4t$



solution via KVL, KCL, Ohm:



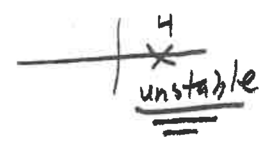
1 KCL = $\boxed{x = u - y}$ ← this was easy to get! (Not always the case!!!)

2 KVL = $\boxed{(s+1)\overbrace{(u-y)}^{x=u-y} = 6\overbrace{(u-y)}^{x=u-y} + y}$

$$\Rightarrow y\,[\underbrace{s+1-6+1}_{s-4}] = u\,[\underbrace{s+1-6}_{s-5}]$$

$$\Rightarrow \boxed{H = \frac{y}{u} = \frac{s-5}{s-4}} \qquad \dot{y} - 4y = \dot{u} - 5u$$

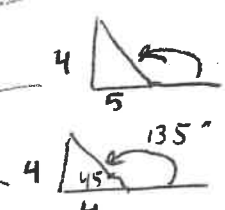
$\hookrightarrow \Phi = s - 4 = 0 \overset{\text{pole}=}{\Longrightarrow}$ pole at $s = 4$



components of y that can be computed using MOTF!

$$\boxed{y_{MOTF} = 4\,H(0) + |H(j4)|\cos(4t + \angle H(j4))}$$

$H(0) = \frac{5}{4}$ $\qquad H(j4) = \dfrac{-5 + j4}{-4 + j4}$



$$= \frac{\sqrt{16+25}\; e^{j(180 - \tan^{-1}(\frac{4}{5}))}}{\sqrt{2}\, e^{j135°}}$$

$$= \frac{\sqrt{16+25}}{\sqrt{2}}\, e^{j(180 - \tan^{-1}(\frac{4}{5}) - 135°)}$$

$$\boxed{B = \frac{(-1)(16+64)}{4\,(16+16)}}$$

$U(s) = \frac{4}{s} + \frac{s}{s^2+16} = \frac{5s^2 + 64}{s(s^2+16)}$

$Y(s) = H(s)\,U(s)$

$= \left[\frac{s-5}{s-4}\right]\left[\frac{5s^2+64}{s(s^2+16)}\right]$

$= \frac{A}{s} + \frac{B}{s-4} + \left[\frac{C}{s-j4} + *\right] \Rightarrow \boxed{y(t) = A + Be^{4t} + 2|C|\cos(4t + \angle C)}$

y_{MOTF} $\qquad y_{ss}$ (unstable)

$\boxed{A = \lim_{s \to 0} sY(s) \overset{MOTF}{=} 4\,H(0)}$

$\boxed{B = \lim_{s \to 4} (s-4)\,Y(s) = (s-5)\left[\frac{5s^2+64}{s(s^2+16)}\right]\Big|_{s=4}}$

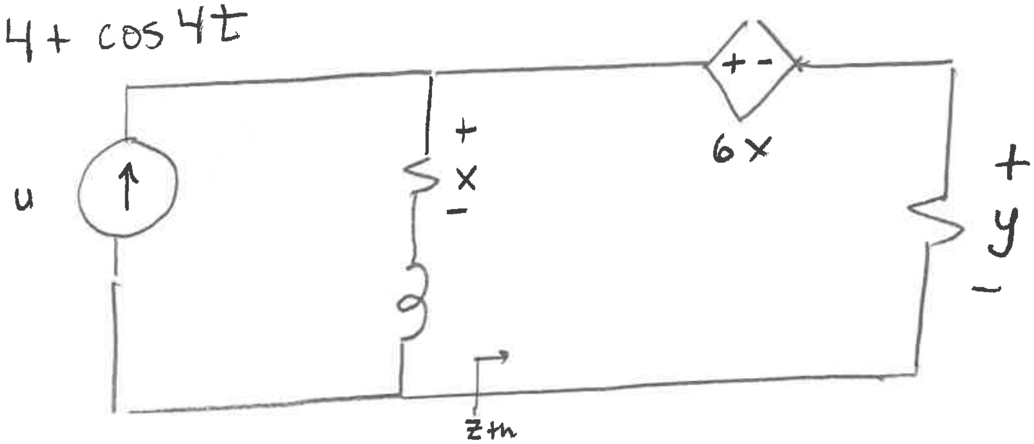
$\boxed{C = \lim_{s \to j4} (s-j4)\,Y(s) \overset{MOTF}{=} \frac{|H(j4)|}{2} e^{j(\angle H(j4))} = |C|\,e^{j\angle C}}$

$\boxed{9|13}$

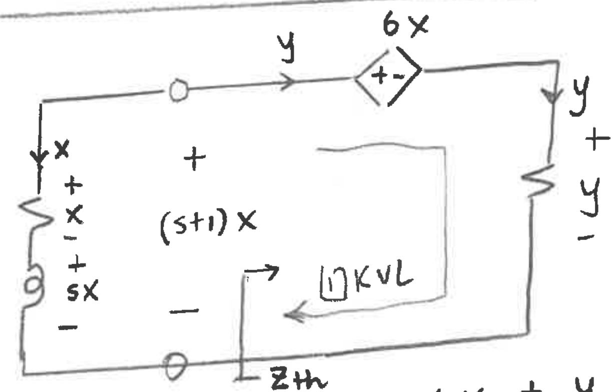
$\boxed{\text{Problem 4}}$ Fall 2024 Final Exam Fri 12/13/24

Determine H, diff eq, y, yss (All R, L = 1)

$u = 4 + \cos 4t$



Alternate solution via Thevenin Equivalent



$\boxed{1}$ KVL: $(s+1)x = 6x + y$

$\Rightarrow x[s+1-6] = y$

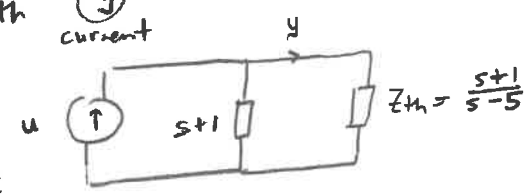
$\Rightarrow x[s-5] = y$

$\Rightarrow x = \frac{y}{s-5}$

$\Rightarrow (s+1)x = \left[\frac{s+1}{s-5}\right]y$

$\Rightarrow Z_{th} = \frac{\overset{\text{voltage}}{(s+1)x}}{\underset{\text{current}}{(y)}} = \frac{s+1}{s-5}$ ← seems complicated but it will yield correct result!

$\Rightarrow$



$\Rightarrow \overset{\text{current division}}{y} = \left[\frac{s+1}{s+1+\frac{s+1}{s-5}}\right]u = \left[\frac{1}{1+\frac{1}{s-5}}\right]u = \left[\frac{s-5}{s-4}\right]u$

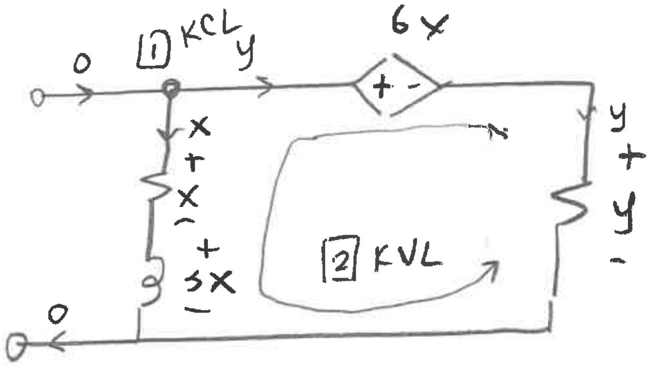
same as before!

$\boxed{8|13}$

Problem 4

Pole Check:

Set $u = 0$ (open)



$\boxed{1}$ KCL = $\boxed{x = -y}$

$\boxed{2}$ KVL = $y = -6x + x + sx$

$= [-6 + 1 + s]\, x \quad \leftarrow x = -y$

$= -[-6 + 1 + s]\, y$

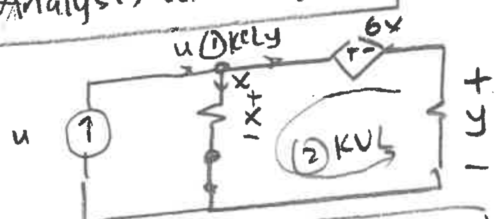
$\Rightarrow y[1 - 6 + 1 + s] = 0$

$\Rightarrow y[s - 4] = 0$

$\Rightarrow \Phi = s - 4 = 0 \quad \leftarrow$ Agrees with what we got in $H(s)$!

Analysis at dc ($s=0$)

$Z_L = sL = 0$ (short)



(1) KCL = $\boxed{x = u - y}$

(2) KVL = $y = -6x + x = -5x = -5(u - y) = -5u + 5y$

$\Rightarrow 4y = 5u \Rightarrow y = \left(\frac{5}{4}\right)u \rightarrow \boxed{H(0) = \frac{5}{4}}$ --- just like what we got ~~before~~!

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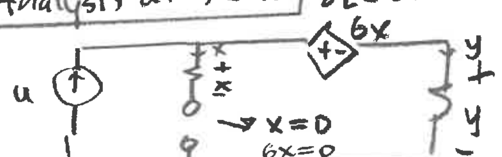
Analysis at $s = \infty$

$Z_L = sL = \infty$ (open)



$\rightarrow y = u \Rightarrow \boxed{H(\infty) = 1}$ --- just like we got before!

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$\boxed{\text{Problem 5}}$

Determine t_s, y_{ss}, y

Note: Show how to compute all coefficients associated with constants, sinusoids, & instabilities in y

Note: You are expected to do the easy calculations.

$$H(s) = \frac{s^2+1}{(s+2)(s^2-6s+25)(s^2+2s+4)}$$

$$u(t) = 5 - 60\sin(t+30^\circ) + 7\cos(100t+45^\circ)$$

Solution:

$H(j1) = 0$ (since $s^2+1\big|_{s=j1} = (j1)^2+1 = -1+1 = 0$)

zero since ~~$H(j1) = 0$!!!~~

$$\boxed{\begin{aligned} y_{MOTF} = {} & 5H(0) - 60|H(j1)|\sin(t+30^\circ + \angle H(j1)) \\ & + 7|H(j100)|\cos(100t+45^\circ + \angle H(j100)) \end{aligned}}$$

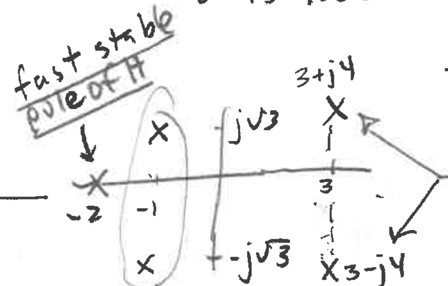
$$H(0) = \frac{(1)}{(2)(25)(4)}$$

$$H(j100) \cong \frac{1}{s^3}\Big|_{s=100e^{j90^\circ}} = \frac{1}{(10^2 e^{j90})^3} = 10^{-6} e^{j(-270^\circ)}$$

poles of H: -2 stable

$3 \pm j4$ unstable

$-1 \pm j\sqrt{3}$ stable



unstable poles of H

slow stable poles of H

$$\Rightarrow t_s = \frac{5}{|\text{Re slow poles}|} = \frac{5}{|-1|} = 5 \text{ sec}$$

Recall: $\boxed{M \leftrightarrow \frac{M}{s}}$

$$\boxed{M\cos(\omega_0 t - \theta) \leftrightarrow M\left[\frac{\cos\theta\, s + \omega_0 \sin\theta}{s^2+\omega_0^2}\right]}$$

$$U(s) = \frac{5}{s} + \frac{num1}{s^2+1} + \frac{num2}{s^2+10^4} = \frac{num}{s(s^2+1)(s^2+10^4)}$$

$$Y = HU = \left[\frac{\cancel{s^2+1}}{(s+2)\underbrace{(s^2-6s+25)}_{3\pm j4}\underbrace{(s^2+2s+4)}_{-1\pm j\sqrt{3}}}\right]\left[\frac{num}{s\cancel{(s^2+1)}\underbrace{(s^2+10^4)}_{\pm j100}}\right]$$

$$= \frac{A}{s} + \frac{B}{s+2} + \left[\frac{C}{s-3-j4} + *\right] + \left[\frac{D}{s+1 \mp j\sqrt{3}} + *\right] + \left[\frac{E}{s-j100} + *\right]$$

$$y = A + Be^{-2t} + 2|C|e^{3t}\cos(4t+\angle C) + 2|D|e^{-t}\cos(\sqrt{3}t + \angle D)$$

$$+ 2|E|\cos(100t + \angle E)$$

stable with $t_s = \frac{5}{1}$ sec

y_{ss} unstable

MOTF

y_{MOTF} $\boxed{A = \lim_{s\to 0} sY(s) = 5H(0)}$ MOTF

$$\boxed{E = \lim_{s\to j100}(s-j100)Y(s) = \frac{7|H(j100)|}{2}e^{j(45^\circ + \angle H(j100))}} = |E|e^{j\angle E}$$

$$\boxed{C = \lim_{s\to 3+j4}(s-3-j4)Y(s)}$$

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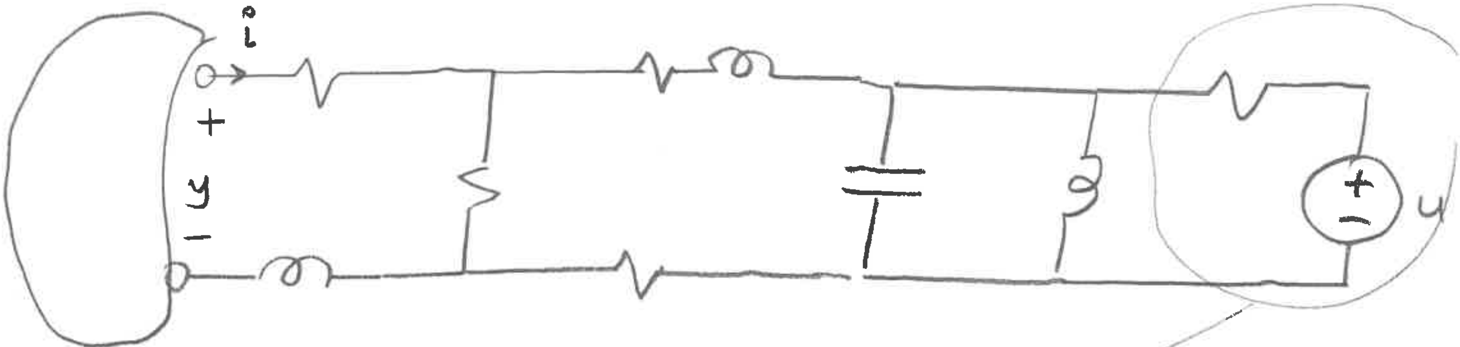
$\boxed{\text{Problem 6}}$

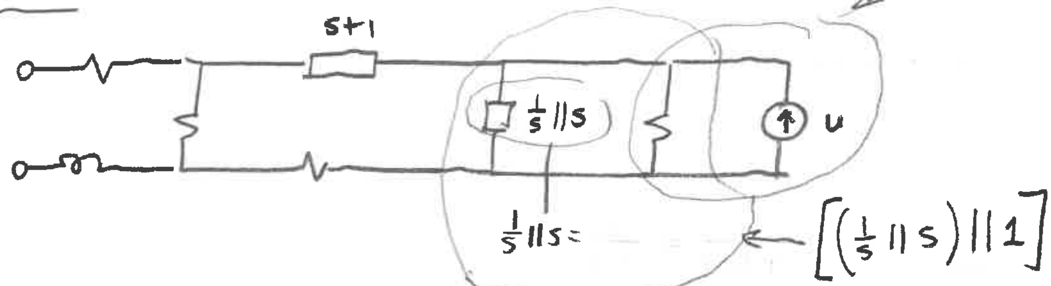
(All R, L, C = 1)

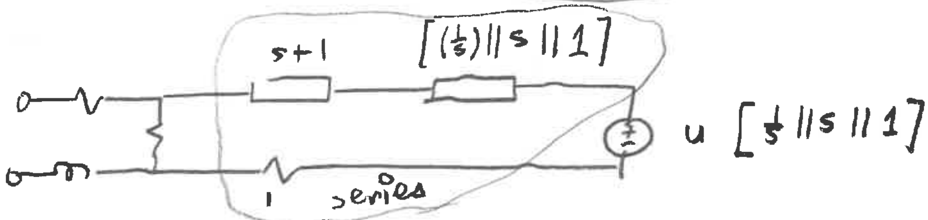
Determine an s-domain thevenin equivalent at y

Must specify Zth & Vth --- emphasize process
... NOT algebra!!!

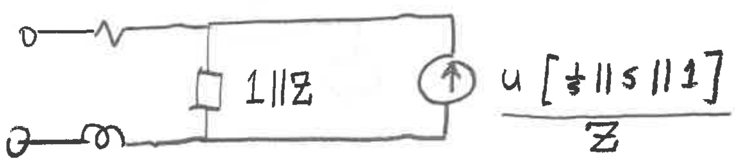


solution:

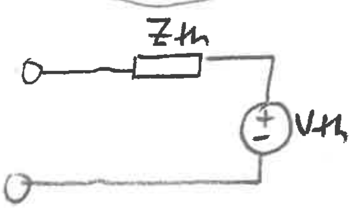




$$\boxed{Z = \overset{s+1+1}{(s+2)} + \left[\tfrac{1}{s}||s||1\right]}$$

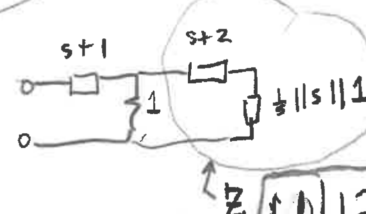


$$\boxed{V_{th} = \frac{u\,[\frac{1}{s}||s||1]}{Z}\cdot[1||Z]}$$



$$\boxed{Z_{th} = s+1 + 1||Z}$$

→



$\boxed{11|13}$

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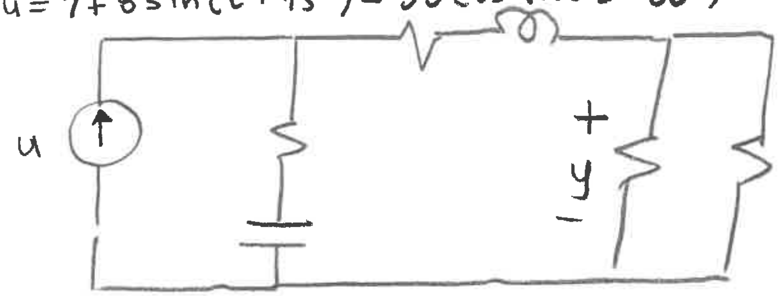
(All R, L, C = 1)

Problem 7

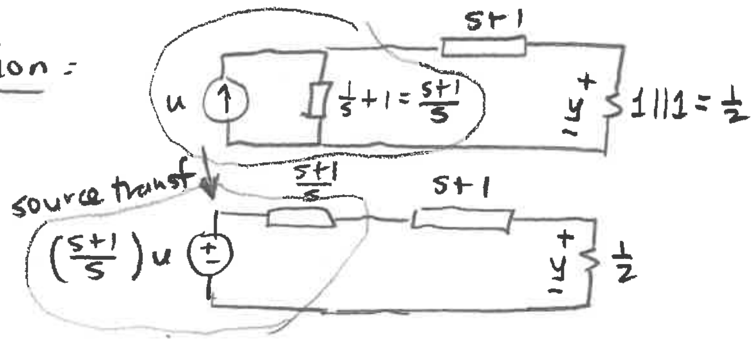
Determine H, y_{ss}, t_s

Note: Determine all coefficients in y_{ss} (approximately).

$u = 7 + 8\sin(t+45^\circ) - 50\cos(100t - 80^\circ)$



solution:



voltage division

$$y \overset{+}{=} \left[\frac{\frac{1}{2}}{\frac{s+1}{s} + s + 1 + \frac{1}{2}}\right]\left(\frac{s+1}{s}\right)u = \left[\frac{\frac{1}{2}(s+1)}{s+1+s^2+s+\frac{1}{2}s}\right]u$$

$$s^2 + \frac{5}{2}s + 1$$

$$\boxed{H = \frac{y}{u} = \frac{\frac{1}{2}(s+1)}{s^2 + \frac{5}{2}s + 1}}$$

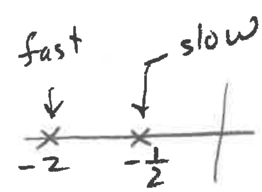
poles ⇝ $s_{1,2} = \dfrac{-\frac{5}{2} \pm \sqrt{\frac{25}{4} - 4}}{2}$

$= -\frac{5}{4} \pm \sqrt{\frac{25}{16} - 1\,\frac{16}{16}} = \frac{9}{16}$

$= -\frac{5}{4} \pm \frac{3}{4}$

$= -\frac{8}{4}, -\frac{2}{4}$

$= -2, -\frac{1}{2}$



$t_s = \dfrac{5}{|\text{slow pole}|} = \dfrac{5}{\frac{1}{2}} = 10\text{ sec}$

NOTF

$$\boxed{\begin{aligned} y_{ss} = {} & 7H(0) \\ & + 8|H(j1)|\sin(t + 45^\circ + \angle H(j1)) \\ & - 50|H(j100)|\cos(100t - 80 + \angle H(j100)) \end{aligned}}$$

$H(0) = 1/2$ (dc gain)

$$H(j1) = \frac{\frac{1}{2}(j1+1)}{\cancel{1} + \frac{5}{2}j1 + \cancel{1}} = \frac{1}{5}\,\frac{(1+j1)}{j1}$$

$$= \frac{1}{5}\,\frac{\sqrt{2}\,e^{j45^\circ}}{1\,e^{j90^\circ}}$$

$$= \frac{\sqrt{2}}{5}\,e^{j(45-90^\circ)}$$

$$H(j100) \cong \frac{1/2}{s}\Big|_{s=100e^{j90^\circ}} = \frac{1/2}{100e^{j90}} = \frac{1}{200}\,e^{j(-90^\circ)}$$

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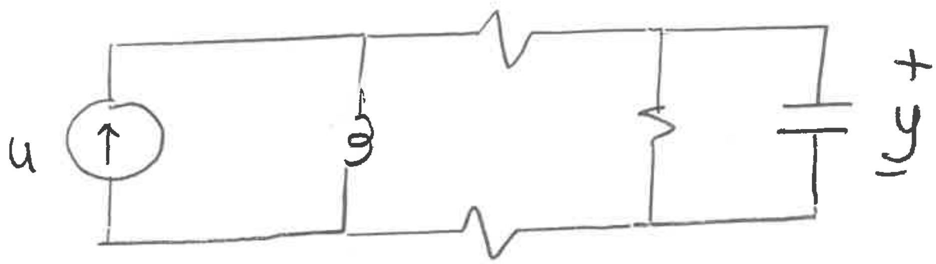
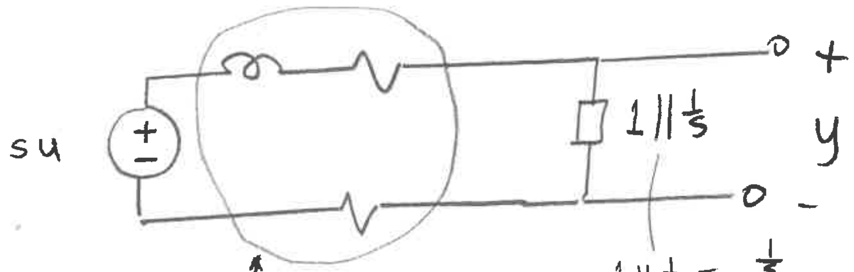
Problem 8

Determine and Plot $|H(j\omega)|$, $\angle H(j\omega)$

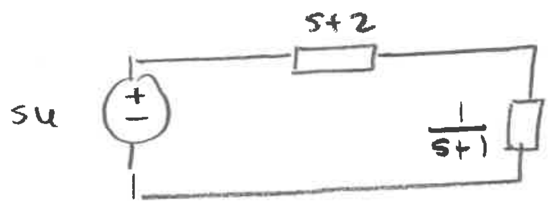
(All R, L, C = 1)



solution via source transf:



$$1 \| \tfrac{1}{s} = \frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{1}{s+1}$$



$$y \overset{\text{voltage division}}{=} \left[\frac{\frac{1}{s+1}}{s+2+\frac{1}{s+1}}\right](su)$$

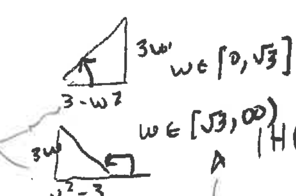
$$= \left[\frac{1}{(s+2)(s+1)+1}\right] su$$

$$= \left[\frac{s}{s^2+3s+3}\right] u$$

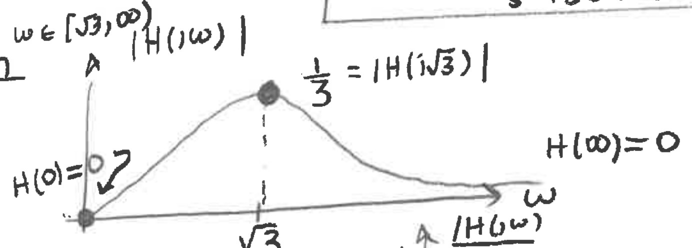
$$\boxed{H(s) = \frac{s}{s^2+3s+3}}$$

$\omega \angle 90°$

$$H(j\omega) = \frac{j\omega}{3-\omega^2 + j3\omega}$$



$$|H(j\omega)| = \frac{\omega}{\sqrt{(3-\omega^2)^2 + (3\omega)^2}}$$



$$\angle H(j\omega) = 90° - \begin{cases} \tan^{-1}\left(\frac{3\omega}{3-\omega^2}\right) & \omega \in [0, \sqrt{3}] \\ 180 - \tan^{-1}\left(\frac{3\omega}{\omega^2-3}\right) & \omega \in [\sqrt{3}, \infty) \end{cases}$$