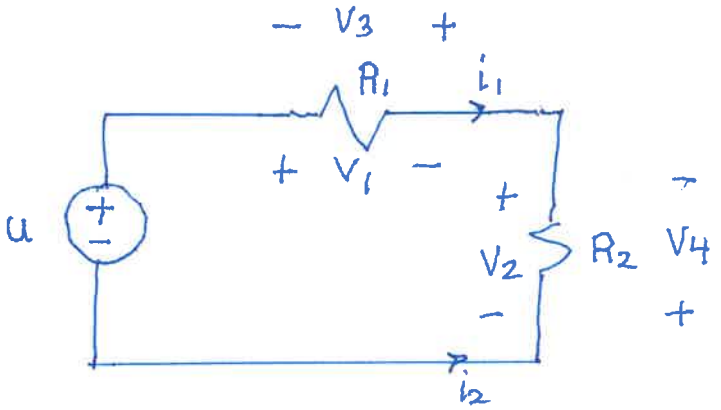
Everything Anyone Would Want to Know About a Voltage Divider

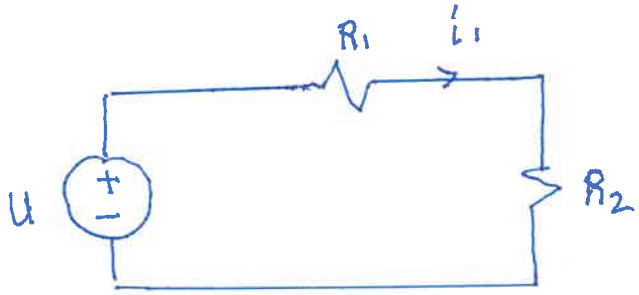


Relate $V_{1,2,3,4}$, $i_{1,2}$ to the input u

I shall solve this many ways... to show you how KVL, KCL, & Ohm can be systematically applied to a simple circuit in many different ways!

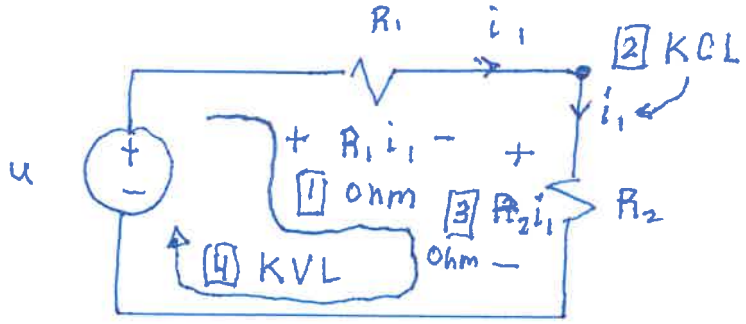
Method 1

Find i_1 first



Lets relate i_1 to u. (3 R_1, R_2)

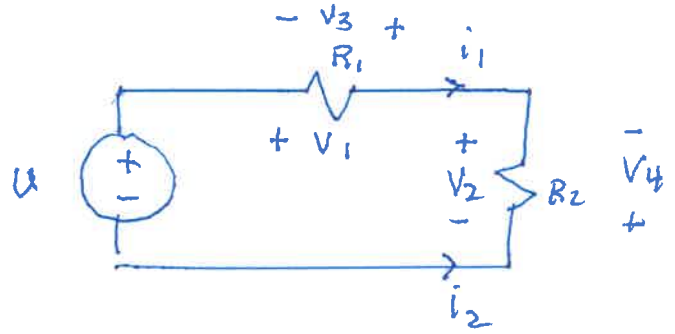
I shall label each step so that you can follow the step-by-step logic!



4 KVL: $u = R_1 i_1 + R_2 i_1$

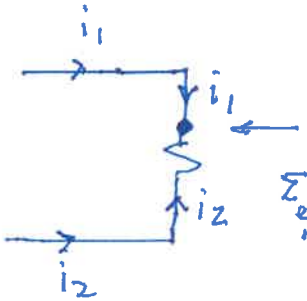
factor i_1: $= (R_1 + R_2) i_1$ $\Rightarrow$ $\boxed{i_1 = \frac{u}{R_1 + R_2}}$

Now that we have i_1, lets find i_2, $V_{1,2,3,4}$ on the original circuit:



From KCL: $i_2 = -i_1$ $\Rightarrow$ $\boxed{i_2 = \frac{-u}{R_1 + R_2}}$

Why?



At this node $\sum i_{\text{entering node}} = i_1 + i_2 = 0$

$\Rightarrow i_2 = -i_1$

From the above:

$V_1 \overset{\text{ohm}}{=} R_1 i_1 \Rightarrow \boxed{V_1 = \left(\frac{R_1}{R_1+R_2}\right) u}$

$V_2 \overset{\text{ohm}}{=} R_2 i_1 \Rightarrow \boxed{V_2 = \left(\frac{R_2}{R_1+R_2}\right) u}$

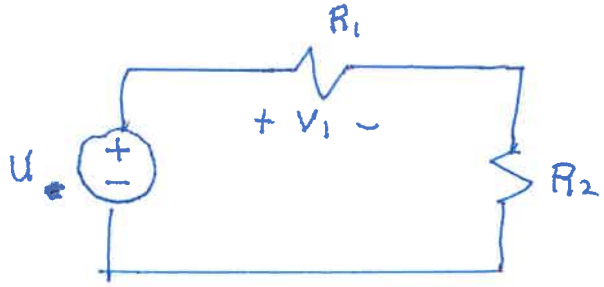
These are referred to as the standard voltage divider formulae!

Remember: Reasons matter!

KVL: $V_3 = -V_1$, $V_4 = -V_2$ $\Rightarrow$ $\boxed{V_3 = -\left(\frac{R_1}{R_1+R_2}\right) u}$ $\boxed{V_4 = -\left(\frac{R_2}{R_1+R_2}\right) u}$

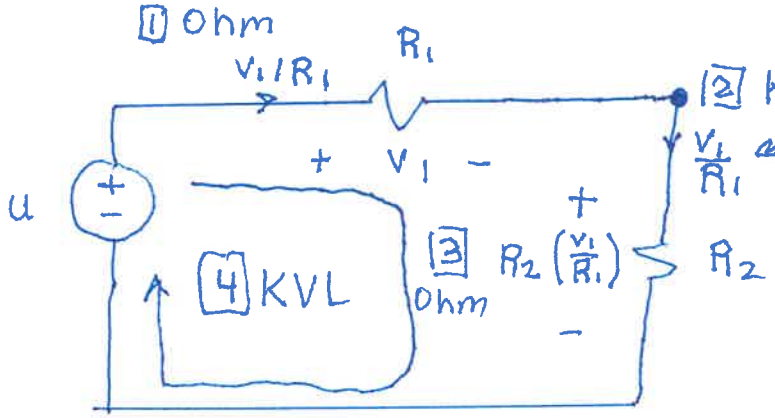
Method 2

Find V_1 first



Lets relate V_1 to u (3 R_1, R_2).

I shall label each step so that you can follow the step-by-step logic!



Note:

Always Draw circuit "bigger" so that you can clearly label steps on the circuit & see your work!

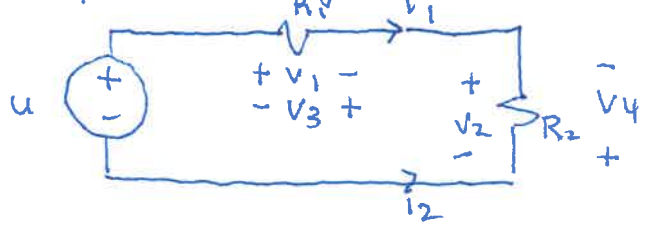
4 KVL = $u = V_1 + R_2\left(\frac{V_1}{R_1}\right)$

↓ factor out V_1

$= (1 + \frac{R_2}{R_1})\, V_1$

$= \left(\frac{R_1 + R_2}{R_1}\right) V_1 \Rightarrow \boxed{V_1 = \left(\frac{R_1}{R_1+R_2}\right) u}$

Looking at the original circuit:



We have $i_1 = \frac{V_1}{R_1}$ ← ohm

$\Rightarrow \boxed{i_1 = \frac{u}{R_1+R_2}}$

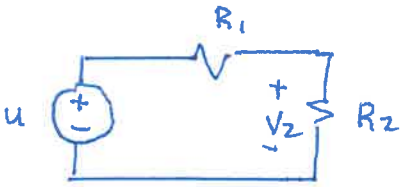
KCL: $i_2 = -i_1 \Longrightarrow$ $\boxed{i_2 = \dfrac{-u}{R_1+R_2}}$

ohm's law: $V_2 = R_2 i_1 \Longrightarrow$ $\boxed{V_2 = \left(\dfrac{R_2}{R_1+R_2}\right) u}$

KVL: $V_3 = -V_1$, $V_4 = -V_2 \Longrightarrow$

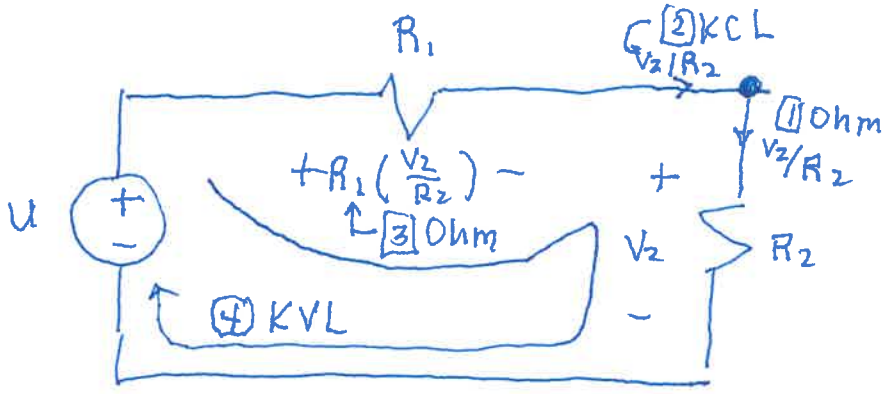
$$\boxed{V_3 = -\left(\frac{R_1}{R_1+R_2}\right) u \qquad V_4 = -\left(\frac{R_2}{R_1+R_2}\right) u}$$

Method 3 — Find V_2 first



Let's relate V_2 to u (R_1, R_2).

I'll label my steps!



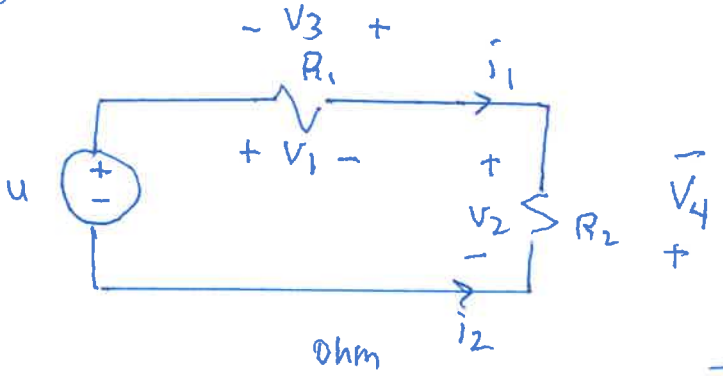
Note: Always draw circuit "bigger" so that you can clearly label steps on the circuit & see your work!

4 KVL: $u = R_1\left(\dfrac{V_2}{R_2}\right) + V_2$

$\overset{\text{factor out } V_2}{=} \left(\dfrac{R_1}{R_2} + 1\right) V_2$

$= \left(\dfrac{R_1+R_2}{R_2}\right) V_2 \Longrightarrow$ $\boxed{V_2 = \left(\dfrac{R_2}{R_1+R_2}\right) u}$

Looking at the original circuit:



we have $\overset{\text{Ohm}}{\downarrow}$ $i_1 = \frac{V_2}{R_2} \Longrightarrow$ $\boxed{i_1 = \frac{u}{R_1 + R_2}}$

KCL: $i_2 = -i_1 \Longrightarrow$ $\boxed{i_2 = \frac{-u}{R_1 + R_2}}$

$\overset{\text{Ohm}}{\downarrow}$ $V_1 = R_1 i_1 \Longrightarrow$ $\boxed{V_1 = \left(\frac{R_1}{R_1 + R_2}\right) u}$

KVL: $V_3 = -V_1$

$V_4 = -V_2$

$$\boxed{\begin{aligned} V_3 &= -\left(\frac{R_1}{R_1 + R_2}\right) u \\ V_4 &= -\left(\frac{R_2}{R_1 + R_2}\right) u \end{aligned}}$$

etc...