Exam Rules

(As discussed in class & "formally announced")

Exam #1
Fri 2/23/24

1. One $8\frac{1}{2}$" x 11" sheet permitted; otherwise closed books, closed notes, closed phones, open minds!

2. Calculator permitted (no computer!!; no phone!!!)

3. NO PHONES! Phones should not be visible at all! A visible phone will result in a zero grade for the exam!

4. Write full name legibly on each page provided.

5. Please show all work. Write your solutions on the pages provided. (No other pages should be used!)

6. Clearly label voltages and currents on circuits provided.

7. Use variables provided! (No additional variables!)

8. Unreadable work will receive no credit.

9. Please place important equations and answers within boxes.

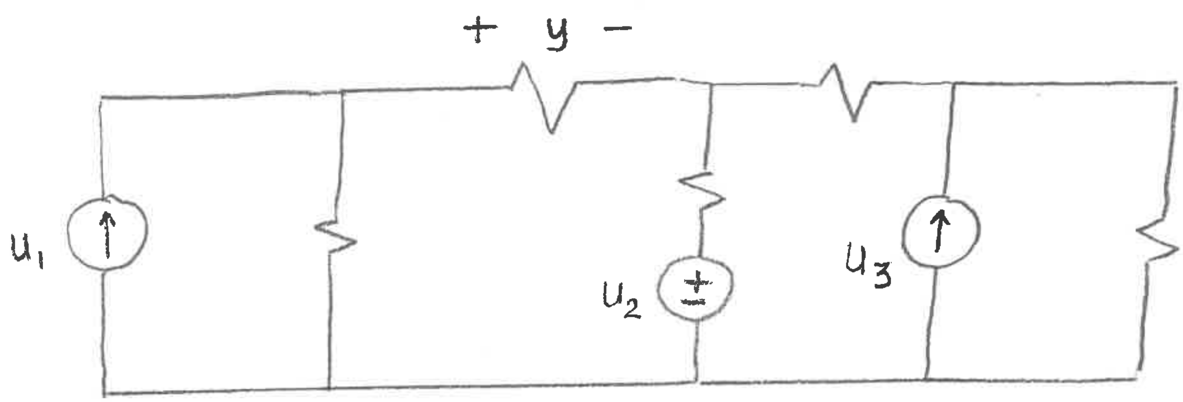
10. Please turn in solutions to me at end of period!

11. PLEASE DO NOT CHEAT!!!

Problem # 1

Relate y to u_1, u_2, u_3

(All $R = 1$)



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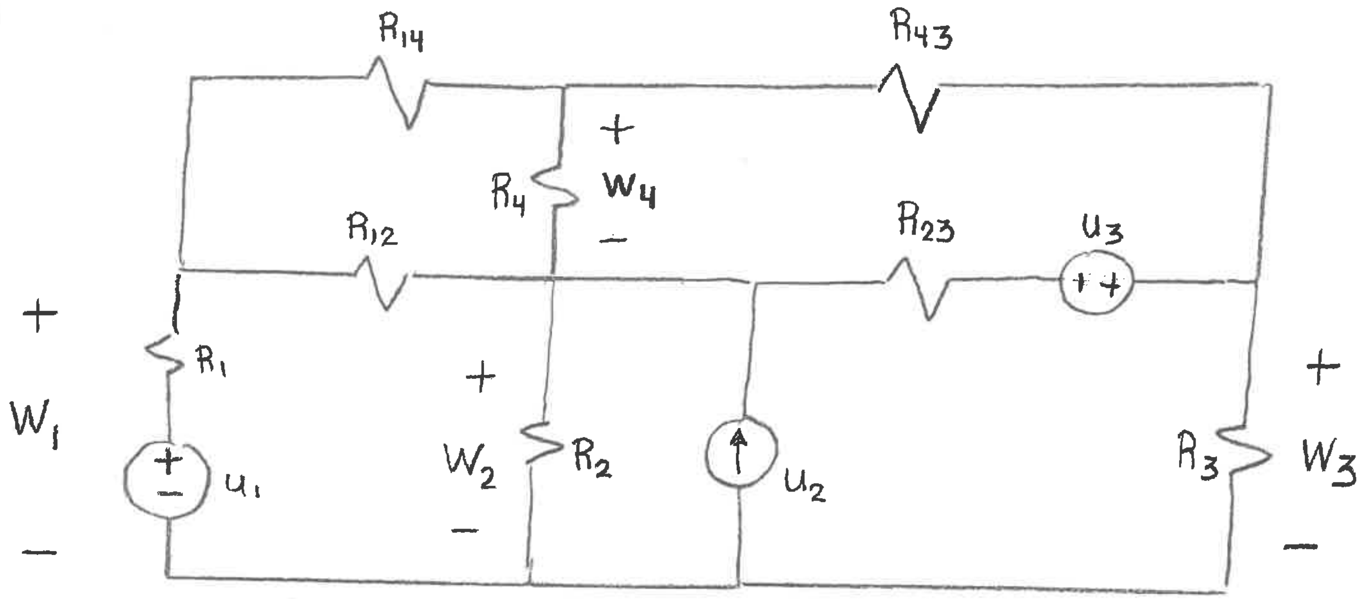
Problem # 2

Relate W_1, W_2, W_3, W_4 to u_1, u_2, u_3 (Nodal Analysis)



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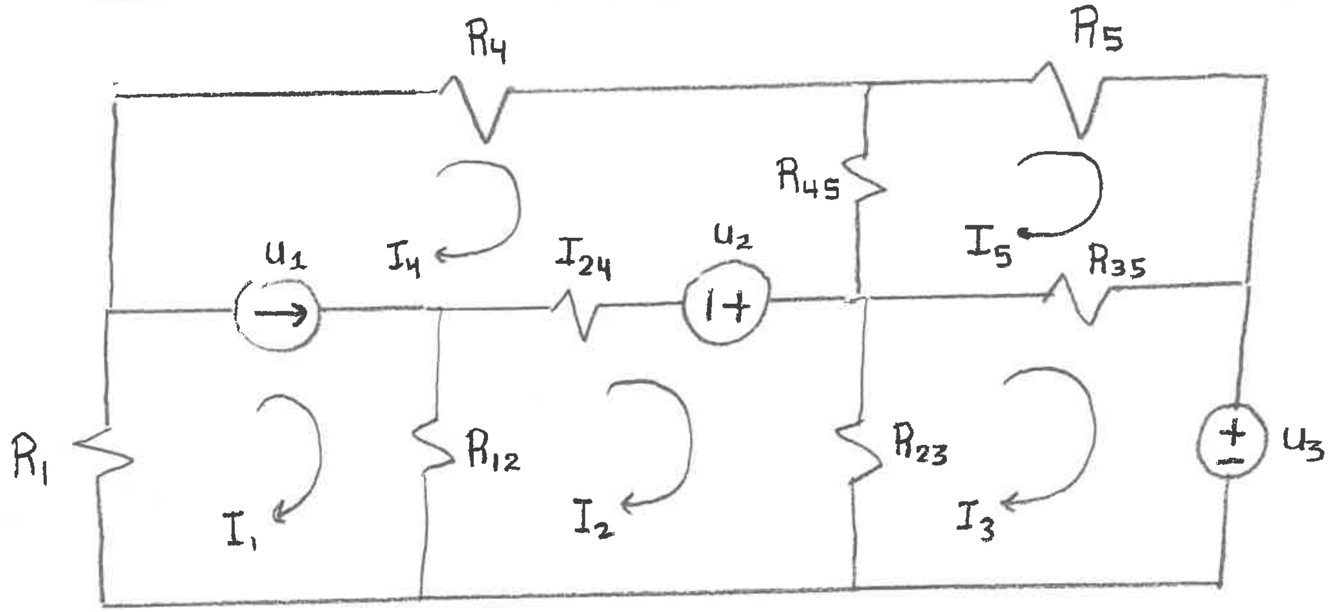
Problem # 3

Relate I_1, I_2, I_3, I_4, I_5 to u_1, u_2, u_3 (Mesh Analysis)



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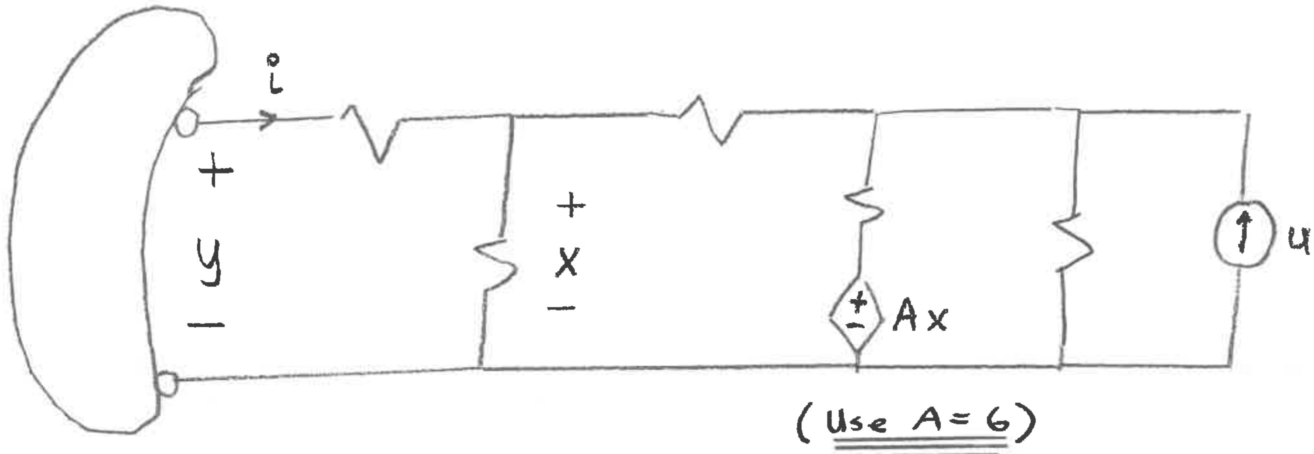
Problem #4

Find a Thevenin equivalent at y

(Thevenin Equiv @ y)

(All R = 1)

(looking rightward)



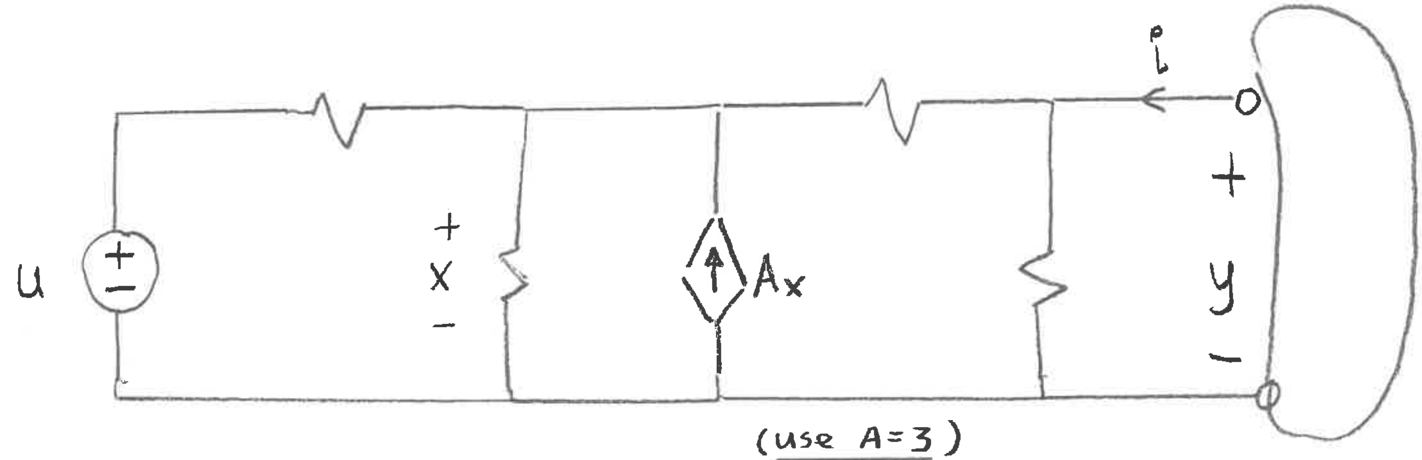
(Use A = 6)

Problem # 5

Find a Thevenin equivalent at y

(Thevenin Equiv @ y)

(All R = 1) (looking leftward)



(use A = 3)

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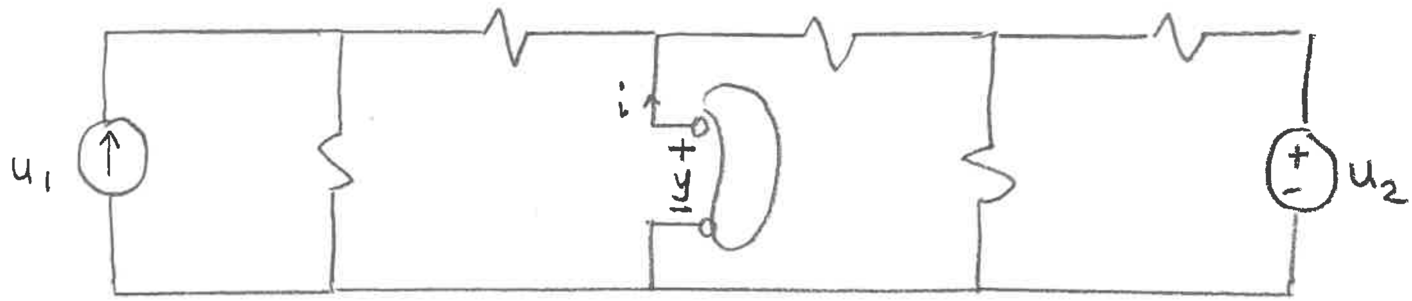
Problem # 6

Find a Thevenin equivalent at y

(Thevenin Equiv @ y)

(All $R = 1$)



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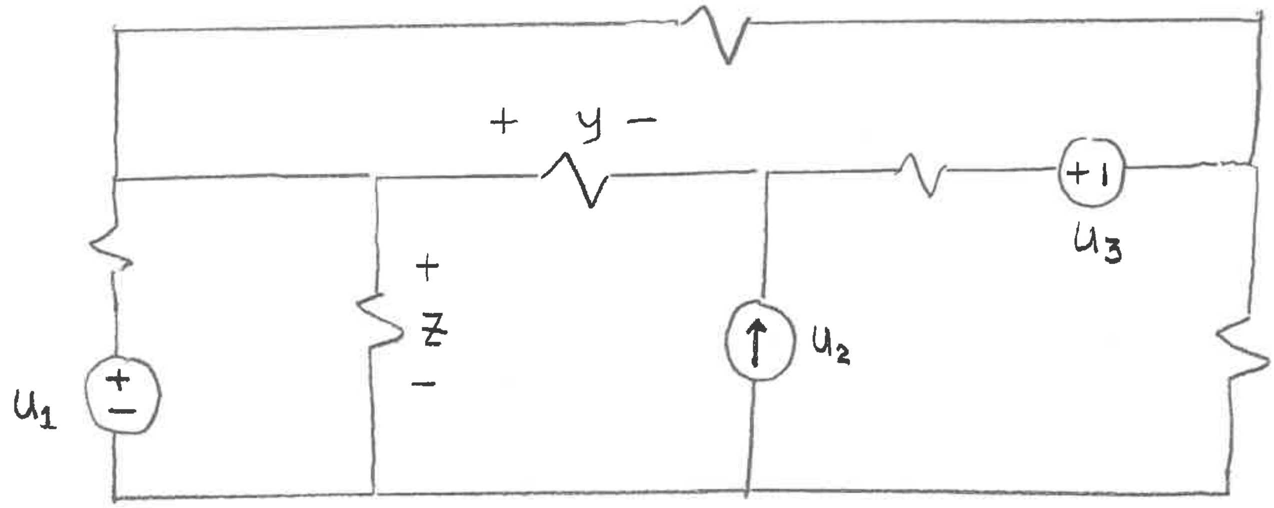
Problem #7

Relate y, z, u_1, u_2, u_3

(All R = 1)

(Introduction of z)



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Solutions

to Exam #1

Fri 2/23/24

Dr. A.A. Rodriguez,

(19 pages follow)

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Problem #1

Relate y to u_1, u_2, u_3

(All $R = 1$)

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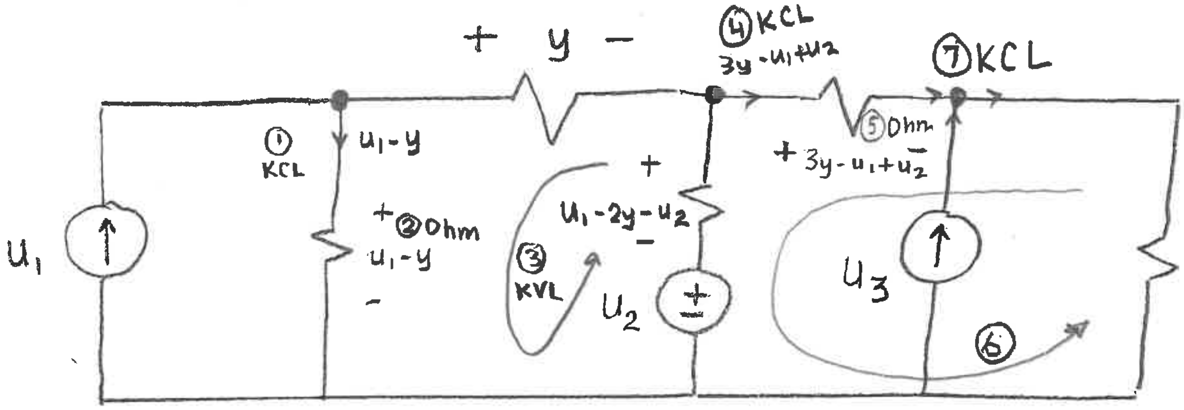


$+$ ⑥ KVL

$u_1 - 3y - u_2$
$+ u_1 - 2y - u_2$
$+ u_2$
$-$ $\|$

$2u_1 - 5y - u_2$

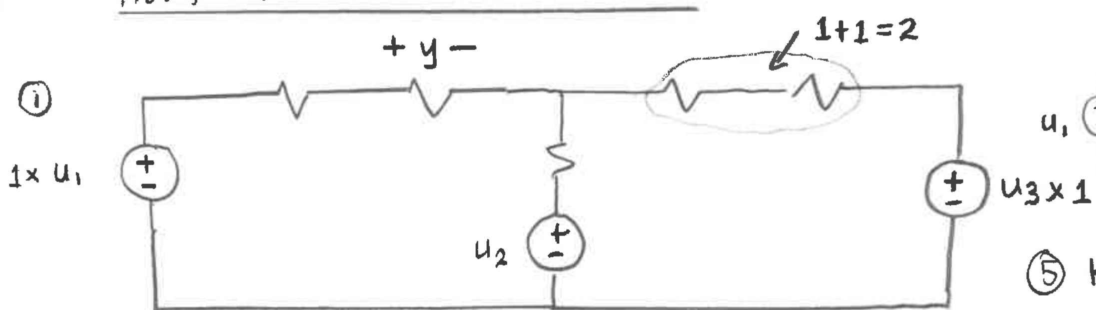
⑦ KCL:

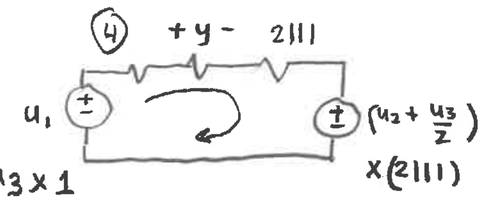
$$\boxed{\left(\frac{3y - u_1 + u_2}{1}\right) + u_3 = \left(\frac{2u_1 - 5y - u_2}{1}\right)}$$

$\xrightarrow{\text{algebra}}$ $8y = 3u_1 - 2u_2 - u_3 \Rightarrow \boxed{y = \left(\frac{3}{8}\right)u_1 + \left(\frac{-2}{8}\right)u_2 + \left(\frac{-1}{8}\right)u_3}$

Above was found via KVL, KCL, Ohm.

Now, lets solve via source transf:





⑤ KVL:

$u_1 = y + y + (2\|1)y + \left(u_2 + \frac{u_3}{2}\right)(2\|1)$

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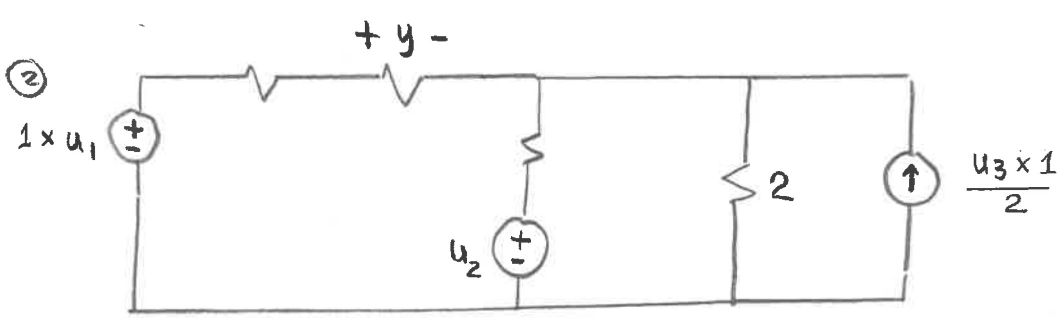
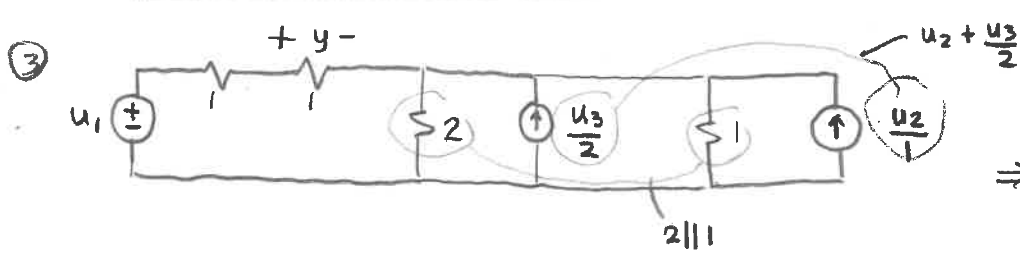
$u_1 = y\left[1 + 1 + \frac{2}{3}\right] + \left[u_2 + \frac{u_3}{2}\right]\left(\frac{2}{3}\right)$

$3u_1 = y\,[\underbrace{3+3+2}_{8}] + \left(u_2 + \frac{u_3}{2}\right)(2)$

$8y = 3u_1 - 2u_2 - u_3$





$\Rightarrow \boxed{y = \left(\frac{3}{8}\right)u_1 + \left(\frac{-2}{8}\right)u_2 + \left(\frac{-1}{8}\right)u_3}$

✓✓✓

Problem # 2

Relate (W_1, W_2, W_3, W_4) to (u_1, u_2, u_3) (Nodal Analysis)

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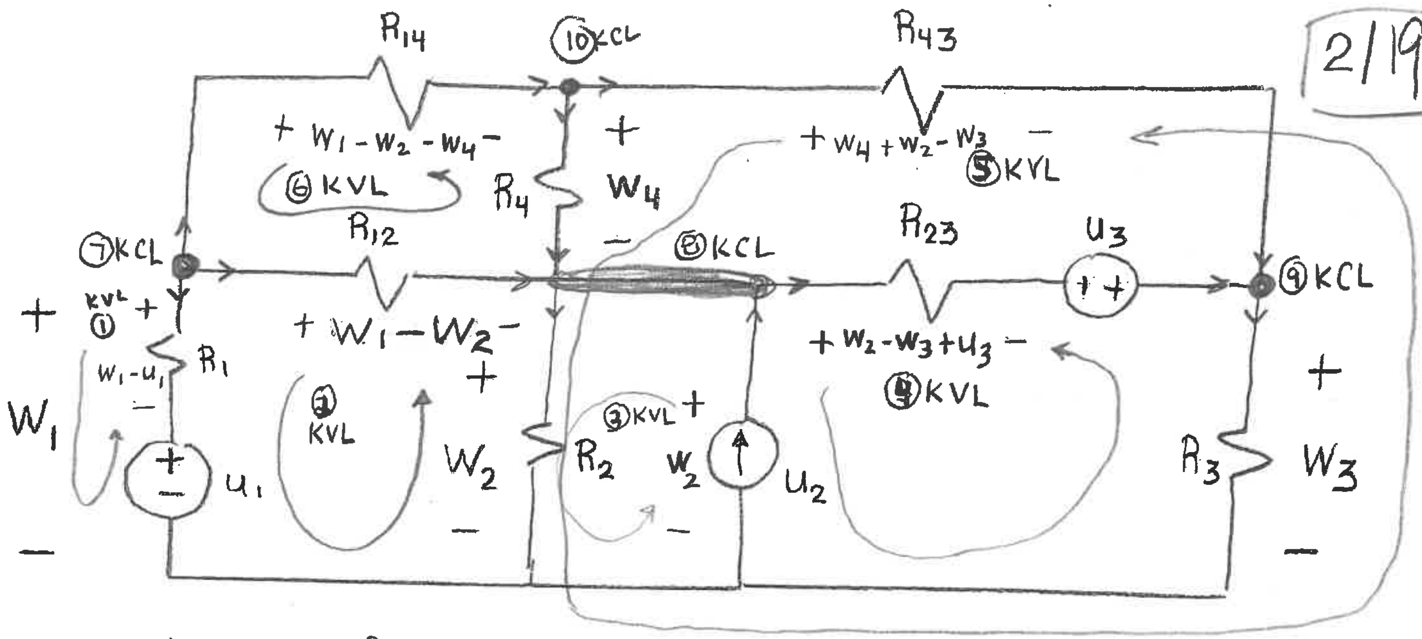

Need 4 eqs in 4 unknowns $W_{1,2,3,4}$.

$(\Sigma\, i_{leaving} = 0)$

⑦ KCL:

$$\boxed{\left(\frac{W_1 - u_1}{R_1}\right) + \left(\frac{W_1 - W_2 - W_4}{R_{14}}\right) + \left(\frac{W_1 - W_2}{R_{12}}\right) = 0}$$

⑧ KCL:

$$\boxed{\left(\frac{W_1 - W_2}{R_{12}}\right) + \left(\frac{W_4}{R_4}\right) + (u_2) = \left(\frac{W_2}{R_2}\right) + \left(\frac{W_2 - W_3 + u_3}{R_{23}}\right)}$$

⑨ KCL:

$$\boxed{\left(\frac{W_2 - W_3 + u_3}{R_{23}}\right) + \left(\frac{W_4 + W_2 - W_3}{R_{43}}\right) = \left(\frac{W_3}{R_3}\right)}$$

⑩ KCL:

$$\boxed{\left(\frac{W_1 - W_2 - W_4}{R_{14}}\right) = \left(\frac{W_4}{R_4}\right) + \left(\frac{W_4 + W_2 - W_3}{R_{43}}\right)}$$

Next, you'll want to put the above in matrix-vector form
– a form that can be fed to your calculator or MATLAB...

Toward the desired matrix-form, we rewrite our 4 eqs as follows:

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$$\left(\frac{1}{R_1}+\frac{1}{R_{14}}+\frac{1}{R_{12}}\right)w_1-\left(\frac{1}{R_{14}}+\frac{1}{R_{12}}\right)w_2+0\,w_3-\left(\frac{1}{R_{14}}\right)w_4=\left(\frac{1}{R_1}\right)u_1$$

$$\left(\frac{1}{R_{12}}\right)w_1-\left(\frac{1}{R_{12}}+\frac{1}{R_2}+\frac{1}{R_{23}}\right)w_2+\left(\frac{1}{R_{23}}\right)w_3+\left(\frac{1}{R_4}\right)w_4=(-1)u_2+\left(\frac{1}{R_{23}}\right)u_3$$

$$0\,w_1+\left(\frac{1}{R_{23}}+\frac{1}{R_{43}}\right)w_2-\left(\frac{1}{R_{23}}+\frac{1}{R_{43}}+\frac{1}{R_3}\right)w_3+\left(\frac{1}{R_{43}}\right)w_4=\left(\frac{-1}{R_{23}}\right)u_3$$

$$\left(\frac{1}{R_{14}}\right)w_1-\left(\frac{1}{R_{14}}+\frac{1}{R_{43}}\right)w_2+\left(\frac{1}{R_{43}}\right)w_3-\left(\frac{1}{R_{14}}+\frac{1}{R_4}+\frac{1}{R_{43}}\right)w_4=0$$

From the above, we get:

$$\overbrace{\begin{bmatrix} \left(\frac{1}{R_1}+\frac{1}{R_{14}}+\frac{1}{R_{12}}\right) & -\left(\frac{1}{R_{14}}+\frac{1}{R_{12}}\right) & 0 & \frac{-1}{R_{14}} \\ \frac{1}{R_{12}} & -\left(\frac{1}{R_{12}}+\frac{1}{R_2}+\frac{1}{R_{23}}\right) & \frac{1}{R_{23}} & \frac{1}{R_4} \\ 0 & \left(\frac{1}{R_{23}}+\frac{1}{R_{43}}\right) & -\left(\frac{1}{R_{23}}+\frac{1}{R_{43}}+\frac{1}{R_3}\right) & \frac{1}{R_{43}} \\ \frac{1}{R_{14}} & -\left(\frac{1}{R_{14}}+\frac{1}{R_{43}}\right) & \left(\frac{1}{R_{43}}\right) & -\left(\frac{1}{R_{14}}+\frac{1}{R_4}+\frac{1}{R_{43}}\right) \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}}^{x}$$

$$=\underbrace{\begin{bmatrix} \frac{1}{R_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{b_1} u_1+\underbrace{\begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}}_{b_2} u_2+\underbrace{\begin{bmatrix} 0 \\ \frac{1}{R_{23}} \\ -\frac{1}{R_{23}} \\ 0 \end{bmatrix}}_{b_3} u_3$$

The above has the matrix-vector form:

$$\boxed{4/19}$$

$$Ax = b_1 u_1 + b_2 u_2 + b_3 u_3$$

If $\left\{\begin{array}{l} Ax_1 = b_1 \\ Ax_2 = b_2 \\ Ax_3 = b_3 \end{array}\right\}$ $\leftarrow$ $\left\{\begin{array}{l} \text{Feed } (A, b_i) \text{ to your calculator} \\ \text{or MATLAB to get the } x_i \end{array}\right\}$

then the desired general solution is given by:

$$X = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = x_1 u_1 + x_2 u_2 + x_3 u_3$$

Why?

$$\begin{aligned} Ax &= A(x_1 u_1 + x_2 u_2 + x_3 u_3) \\ &= (Ax_1) u_1 + (Ax_2) u_2 + (Ax_3) u_3 \\ &= b_1 u_1 + b_2 u_2 + b_3 u_3 \end{aligned}$$

Note:

For $R_i = 1$

$$A = \begin{bmatrix} 3 & -2 & 0 & -1 \\ 1 & -3 & 1 & 1 \\ 0 & 2 & -3 & 1 \\ 1 & -2 & 1 & -3 \end{bmatrix} \quad b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad b_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$b_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

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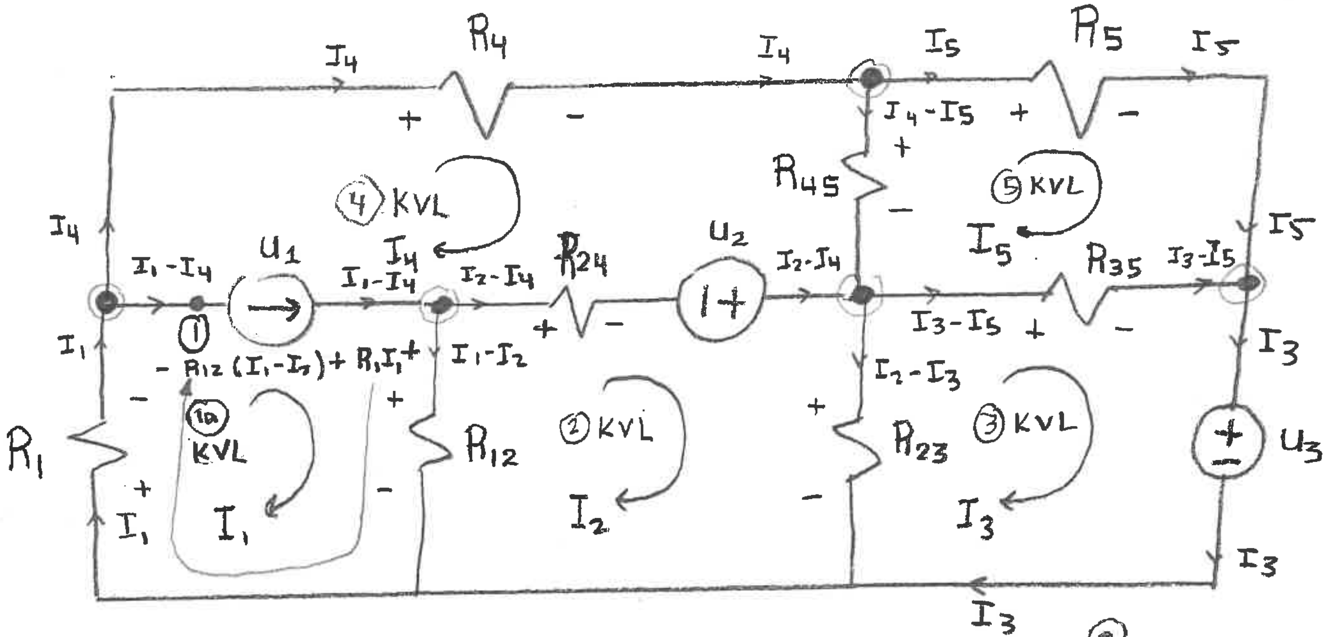
Problem # 3

Relate I_1, I_2, I_3, I_4, I_5 to u_1, u_2, u_3 (Mesh Analysis)



Note: KCL is satisfied at each of the 5 encircled nodes. We shall now get the required 5 eqs in the 5 unknowns $I_{1,2,3,4,5}$.

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① KCL = $\boxed{I_1 - I_4 = u_1}$

② KVL = $\boxed{R_{12}(I_1 - I_2) = R_{24}(I_2 - I_4) - u_2 + R_{23}(I_2 - I_3)}$

③ KVL = $\boxed{R_{23}(I_2 - I_3) = R_{35}(I_3 - I_5) + u_3}$

④ KVL = $\boxed{-R_4 I_4 = R_{45}(I_4 - I_5) + u_2 - R_{24}(I_2 - I_4) + [R_1 I_1 + R_{12}(I_1 - I_2)]}$

⑤ KVL = $\boxed{R_{45}(I_4 - I_5) = R_5 I_5 - R_{35}(I_3 - I_5)}$.

Next, you'll want to put the above in matrix-vector form - a form that can be fed to your calculator or MATLAB...

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The above eqs may be rewritten as follows:

$$(1) I_1 + 0 (I_2) + 0 I_3 + (-1) I_4 = u_1$$

$$(R_{12}) I_1 - (R_{12} + R_{24} + R_{23}) I_2 + R_{23} I_3 + R_{24} I_4 = -u_2$$

$$0 I_1 + (R_{23}) I_2 - (R_{23} + R_{35}) I_3 + 0 I_4 + (R_{35}) I_5 = u_3$$

$$-(R_1 + R_{12}) I_1 + (R_{24} + R_{12}) I_2 + 0 I_3 - (R_4 + R_{24} + R_{45}) I_4 + R_{45} I_5 = u_2$$

$$0 I_1 + 0 I_2 + (R_{35}) I_3 + (R_{45}) I_4 - (R_{45} + R_5 + R_{35}) I_5 = 0$$

From the above, we get the desired matrix-vector form:

$$\begin{array}{cc} A & x \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ R_{12} & -(R_{12} + R_{24} + R_{23}) & R_{23} & R_{24} & 0 \\ 0 & R_{23} & -(R_{23} + R_{35}) & 0 & R_{35} \\ -(R_1 + R_{12}) & (R_{24} + R_{12}) & 0 & -(R_4 + R_{24} + R_{45}) & R_{45} \\ 0 & 0 & R_{35} & R_{45} & \bullet \end{bmatrix} & \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} \end{array}$$

$$-(R_{45} + R_5 + R_{35})$$

$$= \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{b_1} u_1 + \underbrace{\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{b_2} u_2 + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}}_{b_3} u_3$$

The above has the desired matrix-vector form

$$Ax = b_1 u_1 + b_2 u_2 + b_3 u_3$$

Given that we have

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$$A x = b_1 u_1 + b_2 u_2 + b_3 u_3$$

If $\left\{ \begin{array}{l} A x_1 = b_1 \\ A x_2 = b_2 \\ A x_3 = b_3 \end{array} \right\}$

then the general solution is given by:

$$X = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = X_1 u_1 + X_2 u_2 + X_3 u_3$$

Why?

$$\begin{aligned} Ax &= A(x_1 u_1 + x_2 u_2 + x_3 u_3) \\ &= (Ax_1) u_1 + (Ax_2) u_2 + (Ax_3) u_3 \\ &= b_1 u_1 + b_2 u_2 + b_3 u_3 \end{aligned}$$

Note:

For $R_i = 1$, $A = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & -3 & 1 & 1 & 0 \\ 0 & 1 & -2 & 0 & 1 \\ -2 & 2 & 0 & -3 & 1 \\ 0 & 0 & 1 & 1 & -3 \end{bmatrix}$

$b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $b_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $b_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

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Problem # 4

Find a Thevenin equivalent at y

(Thevenin Equiv @ y)

(All $R = 1$) (looking rightward)

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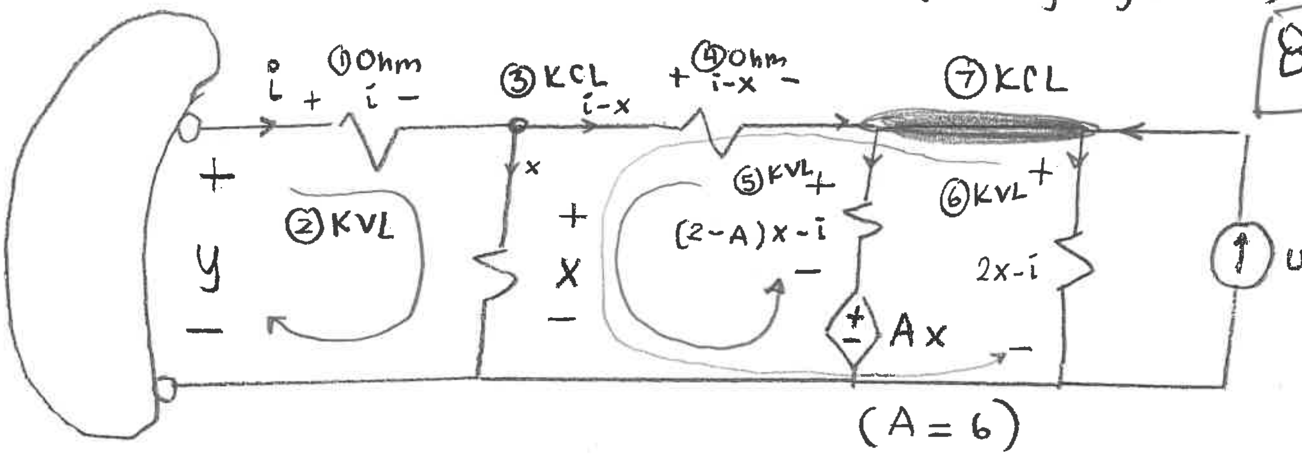
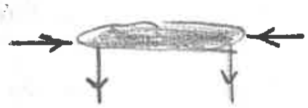


($A = 6$)

② KVL = $\boxed{y = i + x} \Rightarrow x = y - i$

⑦ KCL =



$$\boxed{\overset{\rightarrow}{(i-x)} + \overset{\leftarrow}{(u)} = \overset{\downarrow}{\left(\frac{(2-A)x-i}{1}\right)} + \overset{\downarrow}{\left(\frac{2x-i}{1}\right)}}$$

$\overset{\text{algebra}}{\Longrightarrow}$ $3i + u = \overset{1+2+2}{(5-A)}x$ ($x = y - i$)

$= (5-A)[y-i]$

$\Rightarrow \left(\frac{3}{5-A}\right)i + \left(\frac{1}{5-A}\right)u = y - i$

$\Rightarrow y = \left[1 + \frac{3}{5-A}\right]i + \left[\frac{1}{5-A}\right]u$

$= \left[\frac{5-A+3}{5-A}\right]i + \left[\frac{1}{5-A}\right]u$

$$\boxed{y = \left[\frac{8-A}{5-A}\right]i + \left[\frac{1}{5-A}\right]u}$$

$R_{th} = \frac{8-A}{5-A}$ $V_{th} = \left(\frac{1}{5-A}\right)u$

Note: For $A = 6$, $\boxed{R_{th} = -2 \;,\; V_{th} = -u}$ & $\boxed{y = -2i - u}$

Lets redo the above via source transf:

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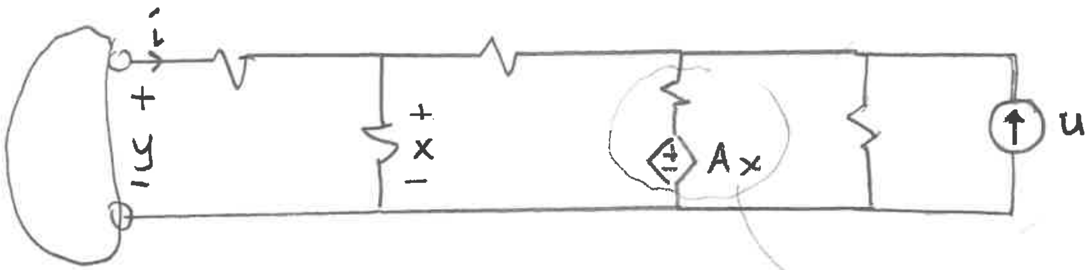
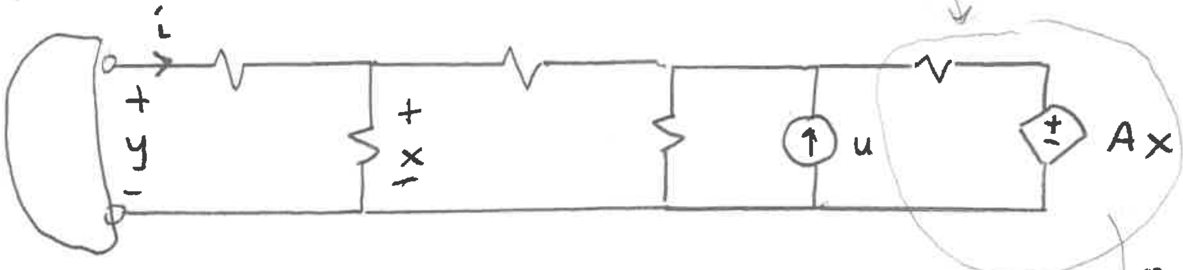


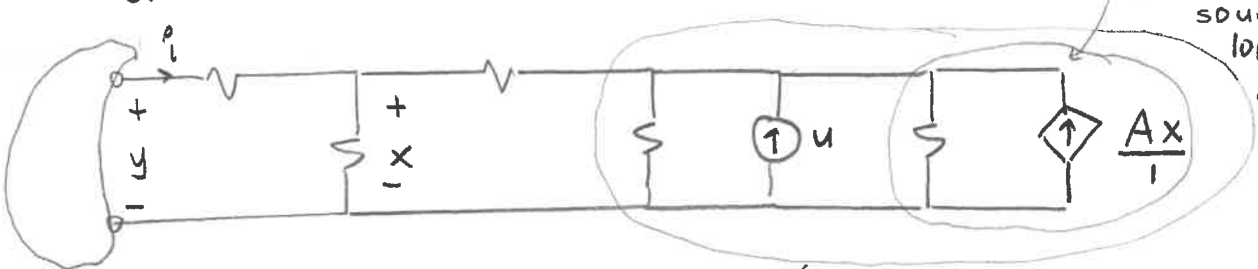
This can be rewritten as:

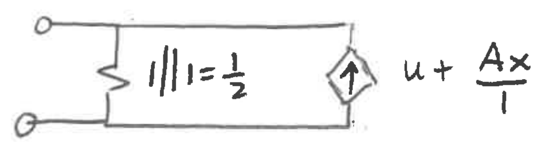
①



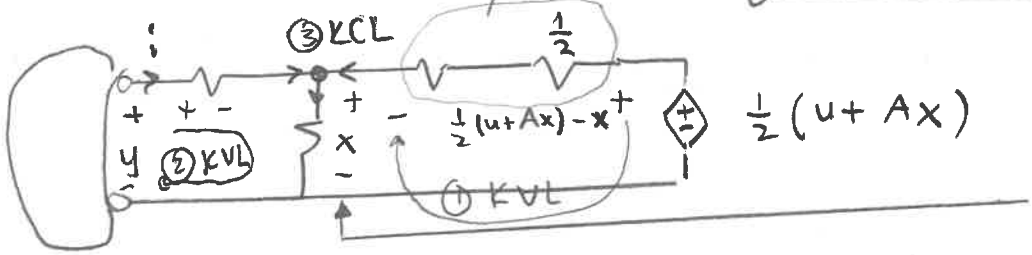
or

② can do source transf on dependent sources so long as you do not destroy the dependent variable x !!!





$1 + \frac{1}{2} = \frac{3}{2}$



Note that dependent variable x was NOT destroyed!

② KVL: $\boxed{y = i + x} \Rightarrow x = y - i$

③ KCL: $\boxed{\vec{i} + \left(\frac{\frac{1}{2}(u+Ax) - x}{3/2} \right) = \left(\frac{x}{1} \right)}$

$\xRightarrow{\text{algebra}} 3i + (u + Ax - 2x) = 3x$

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algebra $\Longrightarrow$

$$x = y - i$$

$$3i + u = (5-A)\,x$$

$$= (5-A)(y-i)$$

$$\Longrightarrow \quad y - i = \left[\frac{3}{5-A}\right] i + \left[\frac{1}{5-A}\right] u$$

$$y = \left[1 + \frac{3}{5-A}\right] i + \left[\frac{1}{5-A}\right] u$$

$$= \left[\frac{5-A+3}{5-A}\right] i + \left[\frac{1}{5-A}\right] u$$

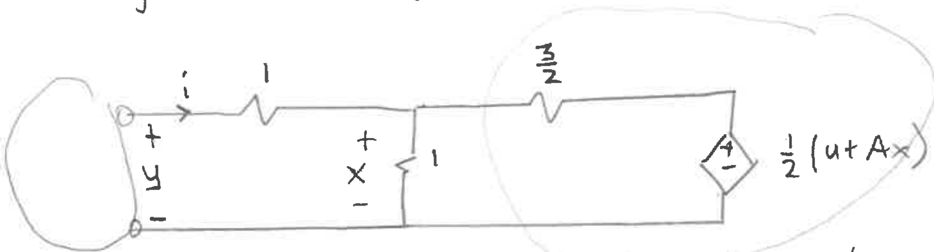
$$\boxed{y = \left[\frac{8-A}{5-A}\right] i + \left[\frac{1}{5-A}\right] u}$$

$$R_{th} = \frac{8-A}{5-A} \overset{A=6}{=} -2 \qquad V_{th} = \left[\frac{1}{5-A}\right] u \overset{A=6}{=} -u$$

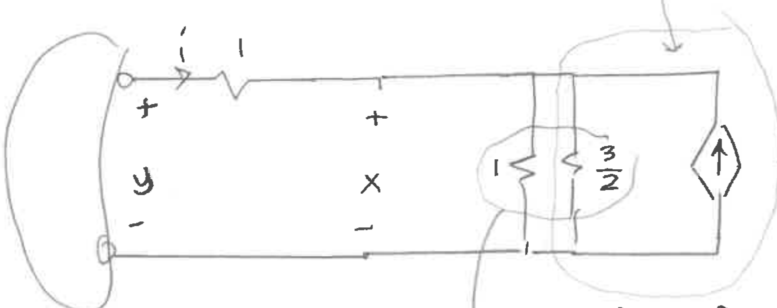
Which is what we obtained earlier! ☺

i.e. $\boxed{y = -2i - u}$ for $A = 6$

To get the above, we started with the following circuit (see previous page)

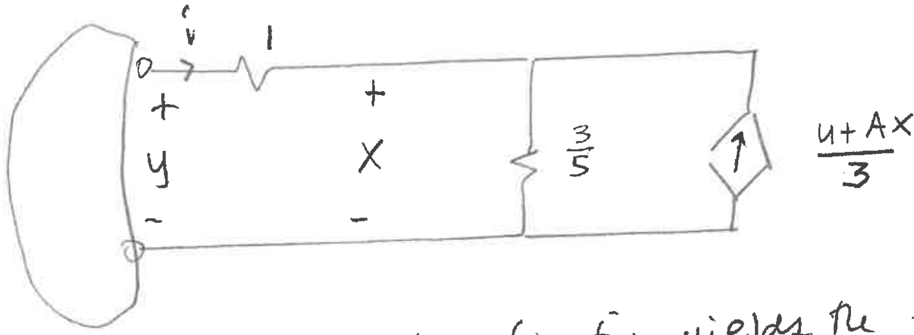


Rather than using KVL, KCL, Ohm, lets do another source transformation:

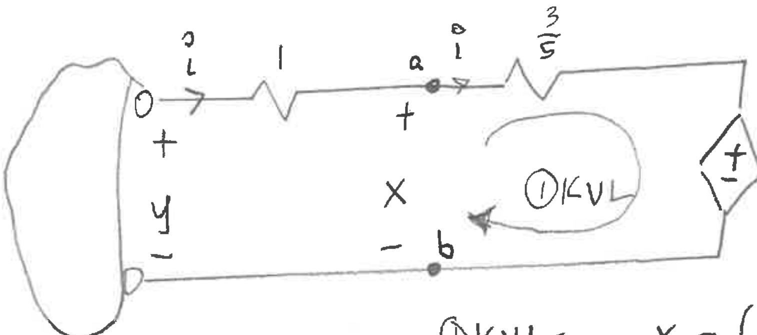


$$\frac{\frac{1}{2}(u+Ax)}{\frac{3}{2}} = \frac{u+Ax}{3}$$

$$1 \| \tfrac{3}{2} = \frac{(1)(\frac{3}{2})}{1+\frac{3}{2}} = \frac{\frac{3}{2}}{\frac{5}{2}} = \frac{3}{5}$$



One final transformation yields the following



$$\left(\frac{3}{5}\right)\left(\frac{u+Ax}{3}\right) = \frac{u+Ax}{5}$$

Note: I have been very careful not to lose the dependent variable x!

① KVL: $x = \left(\frac{3}{5}\right)i + \left(\frac{u+Ax}{5}\right)$

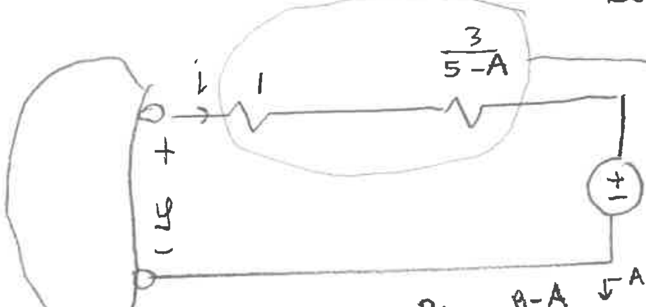
$= \left(\frac{3}{5}\right)i + \frac{u}{5} + \frac{A}{5}x$

$\Rightarrow \quad x\left(1-\frac{A}{5}\right) = \left(\frac{3}{5}\right)i + \frac{u}{5}$

$\Rightarrow \quad x(5-A) = 3i + u$

$\Rightarrow \quad \boxed{x = \left(\frac{3}{5-A}\right)i + \left(\frac{1}{5-A}\right)u}$

This gives us a thevenin equivalent between pts ⓐ & ⓑ.

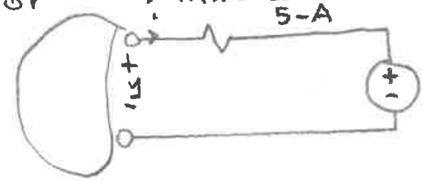


$R_{th} = 1 + \frac{3}{5-A} = \frac{5-A+3}{5-A} = \frac{8-A}{5-A}$

$V_{th} = \frac{u}{5-A}$

or

$R_{th} = \frac{8-A}{5-A} \overset{A=6}{=} -2$



$V_{th} = \frac{u}{5-A} \overset{A=6}{=} -u$

This agrees with what we obtained earlier

--- as it BETTER!

Exam # 1 Fri 2/23/24

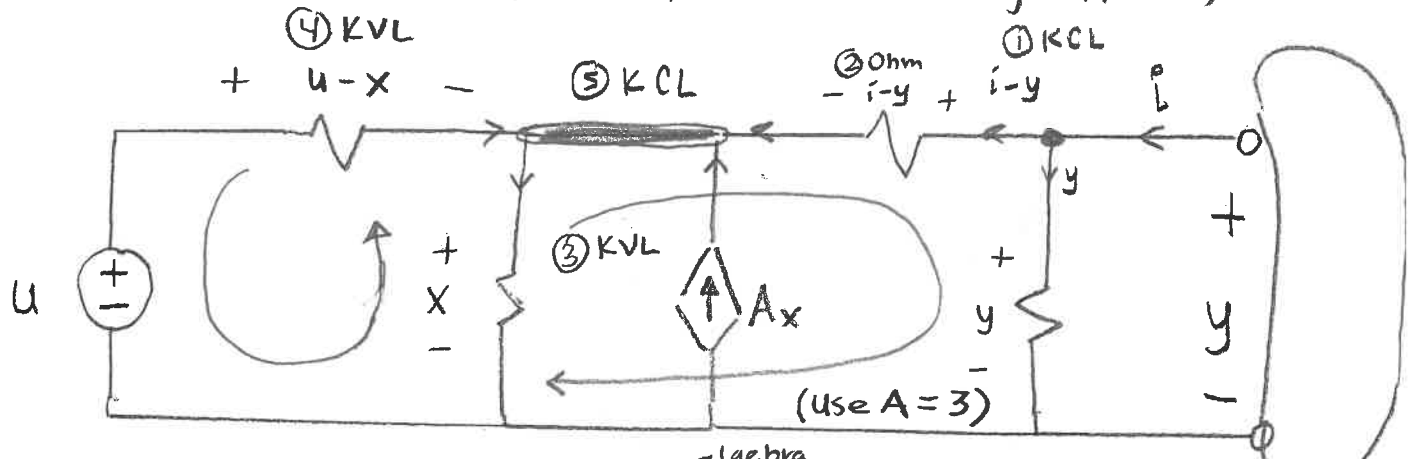
$\boxed{12/19}$

Problem # 5

Find a Thevenin equivalent at y

(Thevenin Equiv @ y)

(All R = 1) (looking leftward)



$$③\ \text{KVL} = \boxed{x = -[i-y] + y \overset{\text{algebra}}{=} -i + 2y}$$

$$⑤\ \text{KCL} = \boxed{\left(\frac{u-x}{1}\right) + (i-y) + (Ax) = \left(\frac{x}{1}\right)}$$

$$\overset{\text{algebra}}{\Longrightarrow} \quad u + i = x[2-A] + y$$

$$= (2y-i)(2-A) + y$$

$$\overset{\text{algebra}}{\Longrightarrow} \quad u + i[\underbrace{1 + 2 - A}_{3-A}] = y[\underbrace{\overbrace{2(2-A)}^{4-2A} + 1}_{5-2A}]$$

$$\Longrightarrow \boxed{y = \left[\frac{3-A}{5-2A}\right] i + \left[\frac{1}{5-2A}\right] u}$$

$$x = 2y - i$$

$$= \left[\frac{6-2A}{5-2A} - 1\right] i + \left[\frac{2}{5-2A}\right] u$$

$$= \left[\frac{6-2A-5+2A}{5-2A}\right] i + [\quad] u$$

$$\boxed{x = \left(\frac{1}{5-2A}\right) i + \left(\frac{2}{5-2A}\right) u} \overset{A=3}{\Longrightarrow} \boxed{x = -i - 2u}$$

$$R_{th} = \frac{3-A}{5-2A} \overset{A=3}{=} 0$$

$$V_{th} = \left(\frac{1}{5-2A}\right) u \overset{A=3}{=} (-1) u$$

This means that

$$\boxed{y = 0i - u = -u}$$

for A = 3

Lets try to understand why we get $R_{th} = 0$ & $V_{th} = -u$ for A = 3...

Exam 1 Fri 2/23/24

Problem # 5

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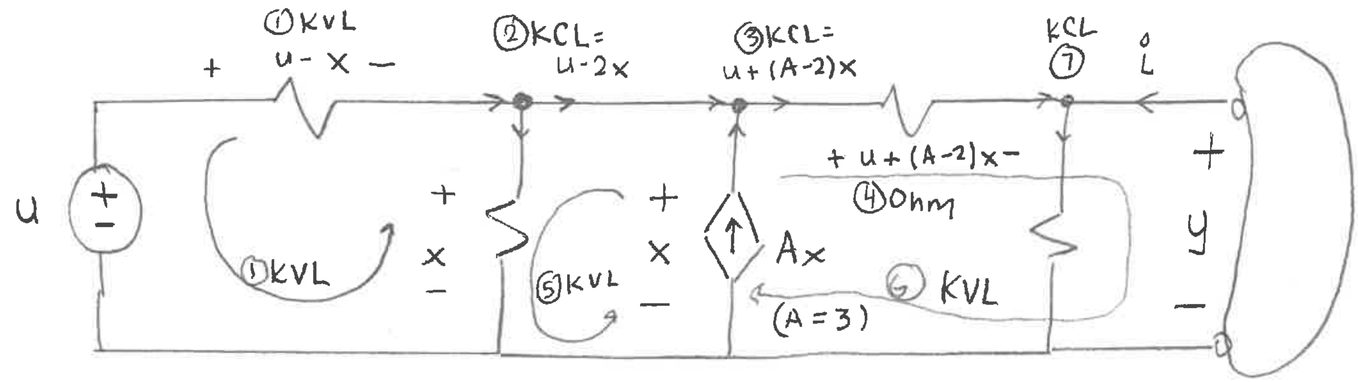
Alternate approach via KVL, KCL, Ohm:



This is a quick way to get to $R_{th} = 0$ for $A = 3$ but it doesn't really show us why!

⑥ KVL: $\boxed{x = u + (A-2)x + y}$

$\overset{A=3}{\Longrightarrow}$ $x = u + x + y$ (x drops out of this eq for A = 3 !!!)

$\Rightarrow$ $\boxed{y = -u}$ $\Rightarrow$ $\boxed{\begin{array}{l} R_{th} = 0 \\ V_{th} = -u \end{array}}$

Note: No additional eq is needed to relate y to i & u !

⑦ KCL: $\boxed{\left(\frac{u+(A-2)x}{1}\right) + \left(i\right) = \left(\frac{y}{1}\right)}$ ← This will be used to solve for x !

$\Rightarrow$ $y = i + u + (A-2)x$

$\overset{A=3}{\Longrightarrow}$ $y = i + u + x$

$\Rightarrow$ $x = y - i - u$

$= -u - i - u$

$\Rightarrow$ $\boxed{x = -i - 2u}$

Lets REALLY try to understant why we get $R_{th} = 0$!

$\Longrightarrow$

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Problem #5

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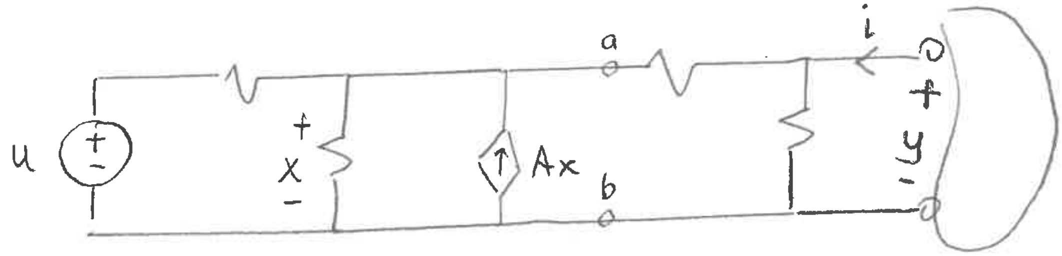
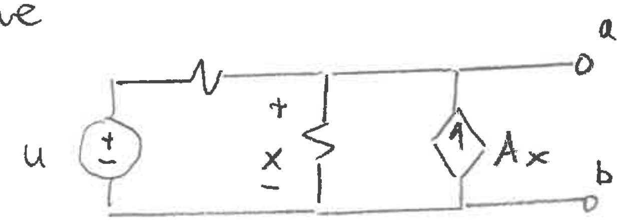
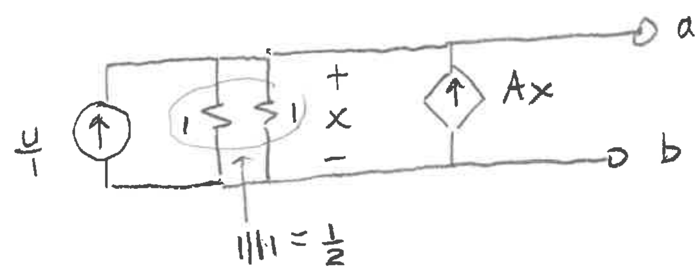


To REALLY understand why $R_{th} = 0$ for $A = 3$, lets find a Thevenin equivalent at terminals ab looking leftward:

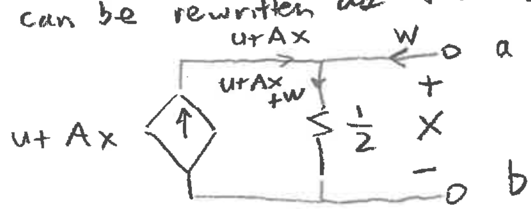
We have



A source transf yields:



This can be rewritten as follows where w is the terminal current



Now lets relate x to w:

$$x \overset{\text{ohm}}{=} \frac{1}{2}\left(\underbrace{u + Ax + w}_{\text{KCL}}\right)$$

$$= \frac{1}{2}u + \frac{1}{2}Ax + \frac{1}{2}w$$

$$2x = u + Ax + w$$

$$(2-A)x = w + u$$

$$\boxed{x = \underbrace{\left(\frac{1}{2-A}\right)}_{R_{thab}} w + \underbrace{\left(\frac{1}{2-A}\right)u}_{v_{thab}}}$$

Exam 1 Fri 2/23/24

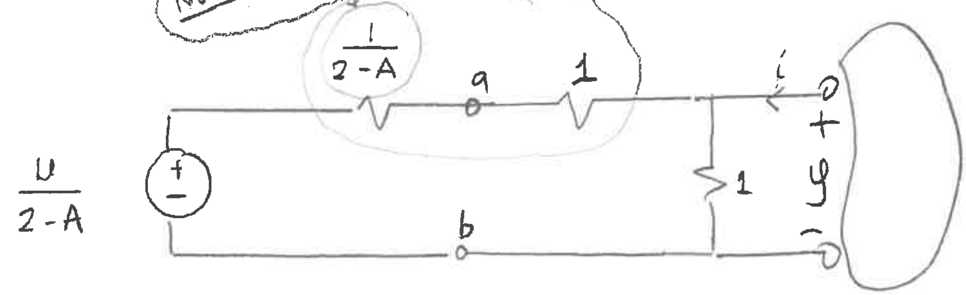
Problem #5

$\boxed{15/19}$

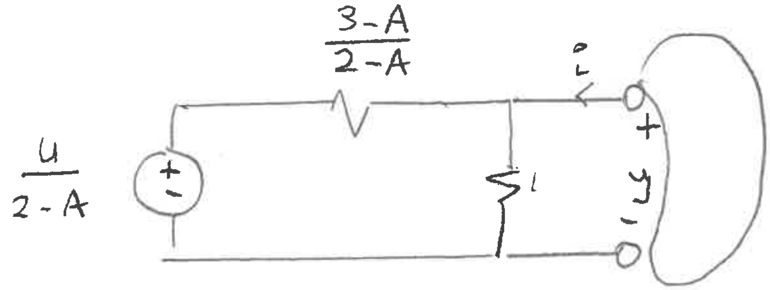
from the above Thevenin equiv between a b looking leftward we got

Note: $\frac{1}{2-A} = -1$ for $A=3$

$$\frac{1}{2-A} + 1 = \frac{1+2-A}{2-A} = \frac{3-A}{2-A}$$



or



From this we now clearly see that

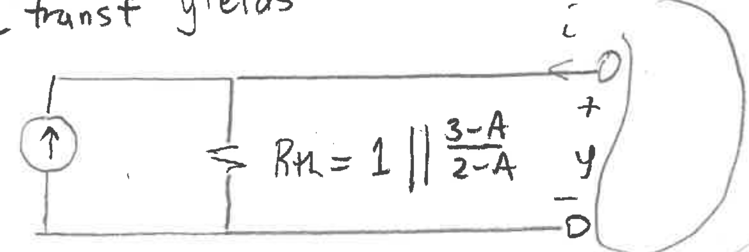
$$R_{th} = 1 \,\|\, \frac{3-A}{2-A}$$

BUT for $A=3$, this becomes

$$R_{th} = 1 \,\|\, 0 = 0 \;!$$

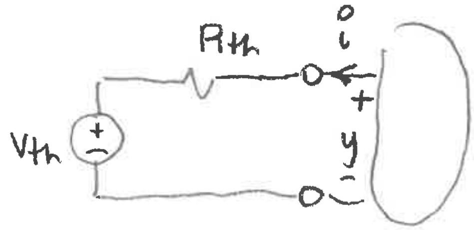
One more source transf yields

$$\left(\frac{u}{\cancel{2-A}}\right)\left(\frac{\cancel{2-A}}{3-A}\right) = \frac{u}{3-A}$$



What we got earlier!

or finally



where

$$R_{th} = 1 \,\|\, \frac{3-A}{2-A} \overset{\text{after algebra}}{=} \frac{3-A}{5-A}$$

$$V_{th} \overset{\text{after algebra}}{=} \frac{u}{5-A}$$

For $A = 3$, we specifically have the following:

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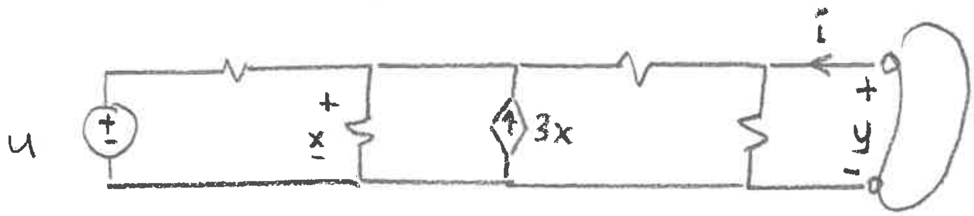
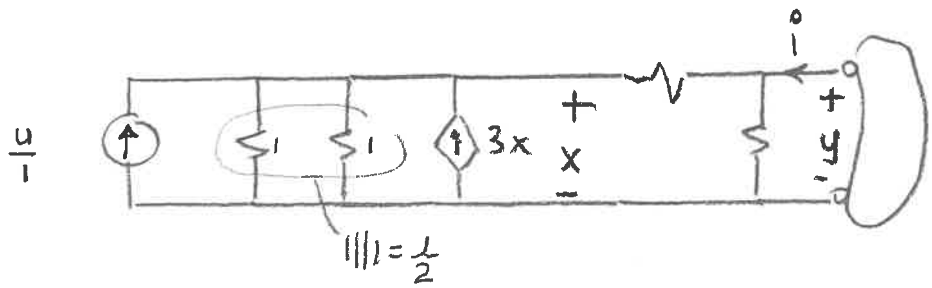
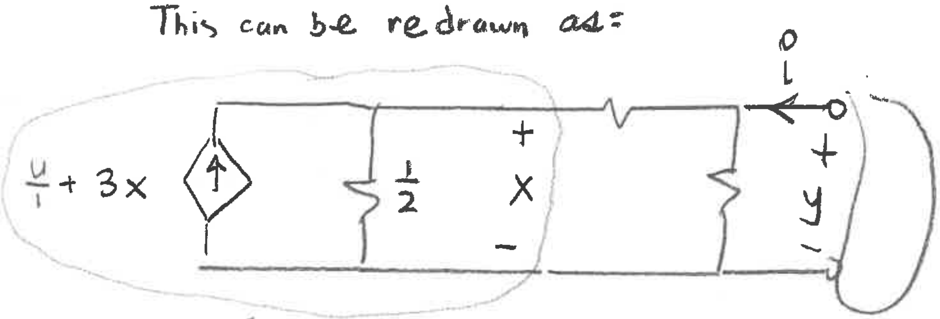


Noting that voltage across 3x current is x, lets use source transf on the left side of the circuit:

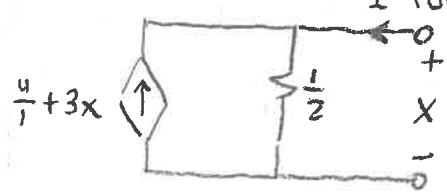


This can be redrawn as:



lets find a Thevenin equiv. for this!

I (do not confuse I with i !!!)



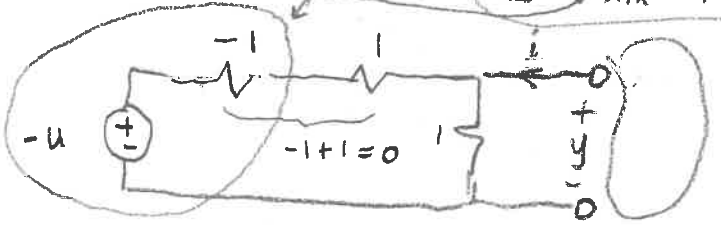
$$\Rightarrow \quad x \overset{\text{ohm}}{=} \frac{1}{2}\left(I \overset{\text{KCL}}{+} \frac{u}{1} + 3x\right)$$

$$= \frac{1}{2} I + \frac{1}{2} u + \frac{3}{2} x$$

$$\Rightarrow \quad -\frac{1}{2} x = \frac{1}{2} I + \frac{1}{2} u$$

$$\Rightarrow \quad x = -I - u$$

$$\Rightarrow \quad R_{th} = -1 \qquad V_{th} = -u$$



$y = 0i + (-u)$

$$\Rightarrow \quad \boxed{y = -u}$$

$$\Rightarrow \quad \boxed{R_{th} = 0 \ \& \ V_{th} = -u}$$

Problem # 6

(Thevenin Equiv @ y)

Find a Thevenin equivalent at y

(All R = 1)

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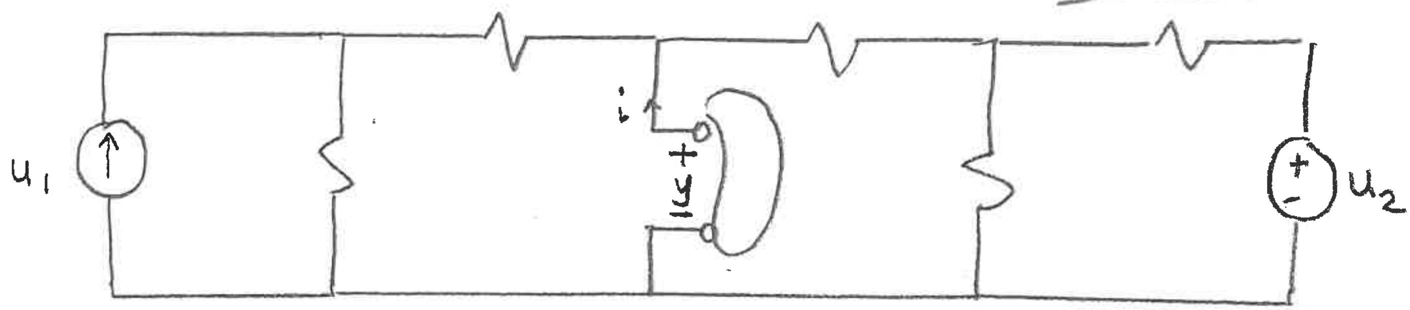
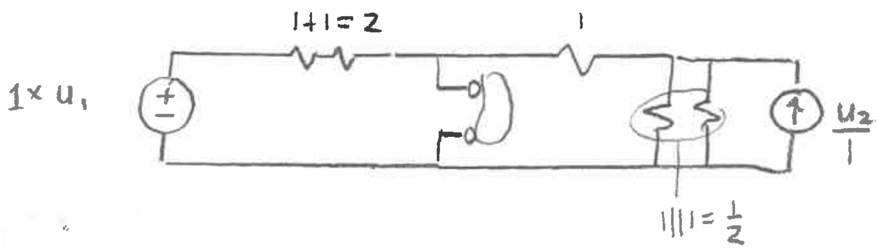
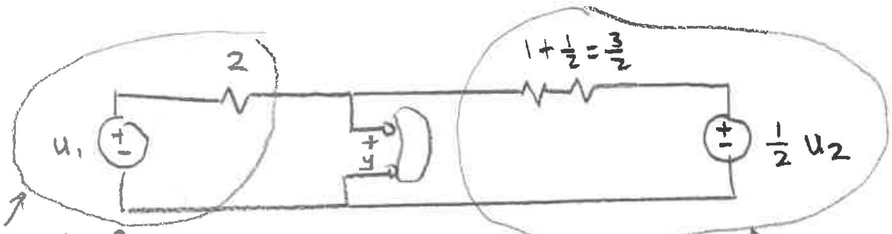


This is best done with source transformations:

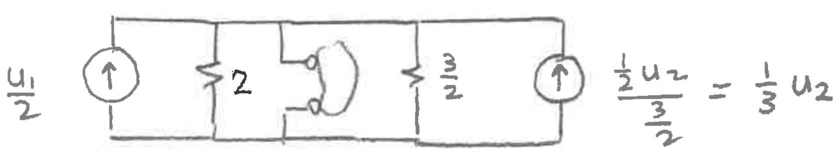
Note: Without source transf you'll need to introduce another variable z in order to use KVL, KCL, Ohm to propagate (y, i) around the cct!

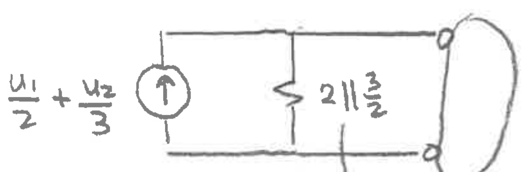


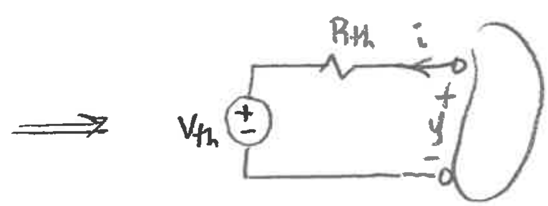


Thev equiv @ y looking leftwards

Thev equiv @ y looking rightwards







$$R_{th} = \frac{2\left(\frac{3}{2}\right)}{2+\frac{3}{2}} = \frac{\frac{6}{2}}{\frac{7}{2}} = \frac{6}{7}$$

$\frac{4}{2}$

$$\boxed{R_{th} = 6/7}$$

$$V_{th} = \left(\frac{u_1}{2}+\frac{u_2}{3}\right)\frac{6}{7}$$

$$\boxed{V_{th} = \frac{3}{7}u_1 + \frac{2}{7}u_2}$$

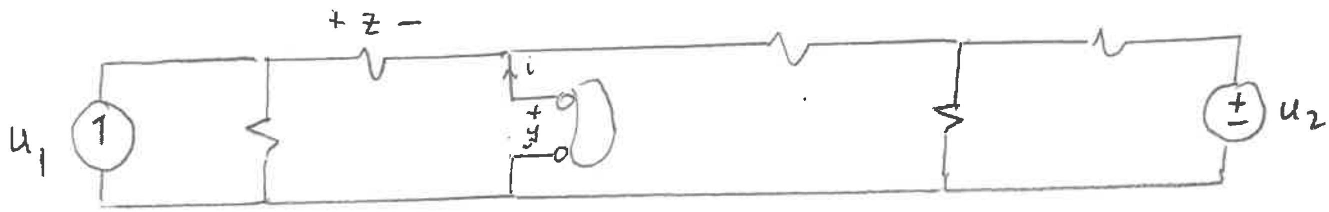
Note: without source transf, we can introduce another variable z & use KVL, KCL, Ohm --- lets do that --- ⟹

Exam #1 Fri 2/23/24

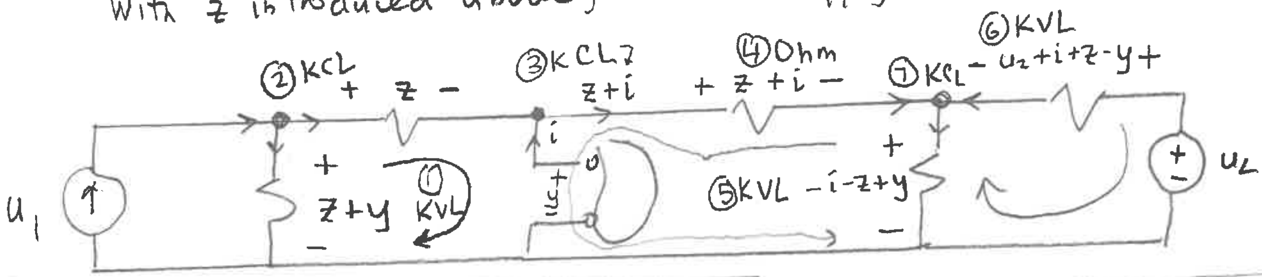
Problem #6

$\boxed{18/19}$

Lets introduce z =



With z introduced above, we can apply KVL, KCL, Ohm as follows:



② KCL = (+Ohm)

$$\boxed{u_1 = \left(\frac{z+y}{1}\right) + \left(\frac{z}{1}\right)}$$

⑦ KCL = (+Ohm)

$$\boxed{\left(\frac{z+i}{1}\right) + \left(\frac{u_2+i+z-y}{1}\right) = \left(\frac{-i-z+y}{1}\right)}$$

$\Rightarrow 2z = u_1 - y$

$\xrightarrow{3\times} 6z = 3u_1 - 3y$

$\Rightarrow 3z = -3i + 2y - u_2$

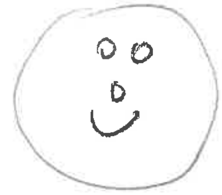
$\xrightarrow{2\times} 6z = -6i + 4y - 2u_2$

$$\Rightarrow 6z = 3u_1 - 3y = -6i + 4y - 2u_2$$

$$\Rightarrow 7y = 6i + 3u_1 + 2u_2$$

$$\Rightarrow \boxed{y = \left(\frac{6}{7}\right)i + \left(\frac{3}{7}\right)u_1 + \left(\frac{2}{7}\right)u_2}$$

$R_{th} = \frac{6}{7}$ $V_{th} = \left(\frac{3}{7}\right)u_1 + \left(\frac{2}{7}\right)u_2$



... just as we got earlier via source transformations

Note:
while introducing z was not too bad, solving via source transf was much "cooler!"

Exam #1 Fri 2/23/24

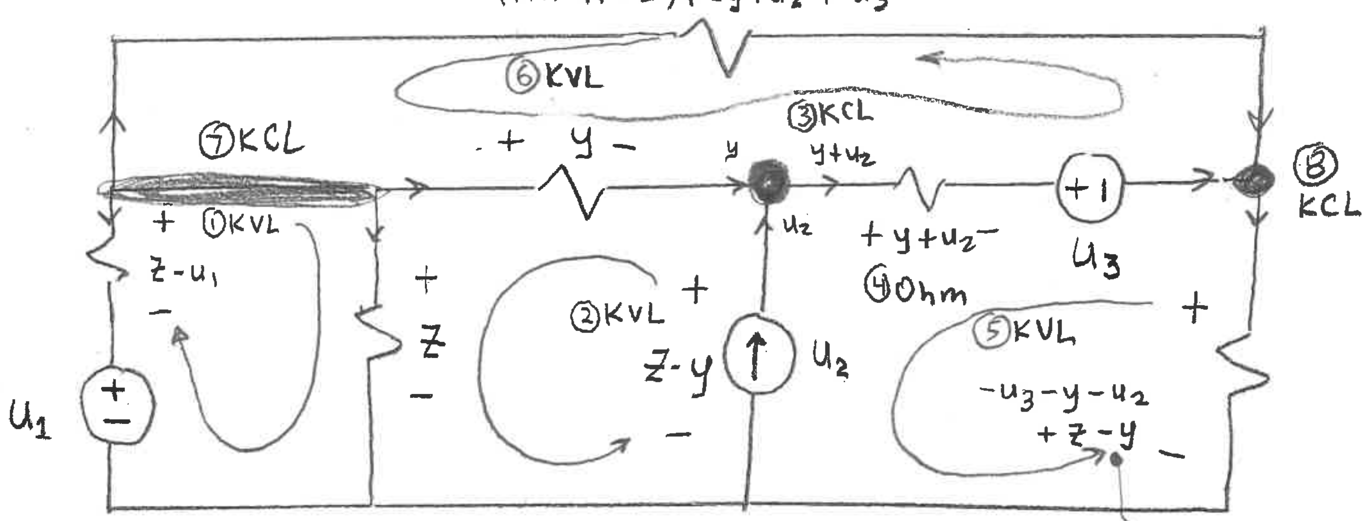
$\boxed{19/19}$

Problem #7

Relate y, z, u_1, u_2, u_3

(Introduction of z)

(All $R = 1$) $+\ 2y + u_2 + u_3\ -$



$= z - 2y - u_2 - u_3$

⑦ KCL =

$$\boxed{\left(\frac{z - u_1}{1}\right) + \left(\frac{2y + u_2 + u_3}{1}\right) + \left(\frac{y}{1}\right) + \left(\frac{z}{1}\right) = 0}$$

$\overset{\text{algebra}}{\Longrightarrow}\quad 2z + 3y = u_1 - u_2 - u_3 \quad \Longrightarrow \quad z = \dfrac{u_1 - u_2 - u_3 - 3y}{2}$

⑧ KCL

$$\boxed{\left(\frac{y + u_2}{1}\right) + \left(\frac{2y + u_2 + u_3}{1}\right) = \left(\frac{z - 2y - u_2 - u_3}{1}\right)}$$

$\overset{\text{algebra}}{\Longrightarrow}\quad z = 5y + 3u_2 + 2u_3$

$$z = 5y + 3u_2 + 2u_3 = \tfrac{1}{2}u_1 - \tfrac{1}{2}u_2 - \tfrac{1}{2}u_3 - \tfrac{3}{2}y$$

$$10y + 6u_2 + 4u_3 = u_1 - u_2 - u_3 - 3y$$

$$13y = u_1 - 7u_2 - 5u_3$$

$$\boxed{y = \left(\tfrac{1}{13}\right)u_1 + \left(\tfrac{-7}{13}\right)u_2 + \left(\tfrac{-5}{13}\right)u_3}$$

Exam #2

Fri 4-19-24

8 Problems (5+3)

(17 pages follow)

EXAM RULES

1) One 8.5" x 11" sheet permitted;
   otherwise closed books, closed notes, open minds!

2) Scientific calculator is permitted; no computers and no phone!

3) **<u>NO PHONES</u> !!!**
   Phones should not be visible at all!
   A visible phone will result in a zero grade for the exam!

4) Write FULL NAME legibly on each page provided.

5) **<u>PLEASE SHOW ALL WORK</u> !!!**
   This is essential to receive partial credit!

6) Please do not submit multiple answers. You must pick an answer!!

7) Write your solutions on the sheets provided.
   No other paper/sheets/pages should be used!

8) Clearly label voltages and currents on the circuits provided.

9) Use the variables provided!
   No additional variables should be used!

10) Unreadable work will receive NO CREDIT.

11) Please place important equations and answers within boxes as we have done in lecture.

12) Please be careful with your algebra, signs, etc.

13) Please turn in your solutions to me at the end of the period!

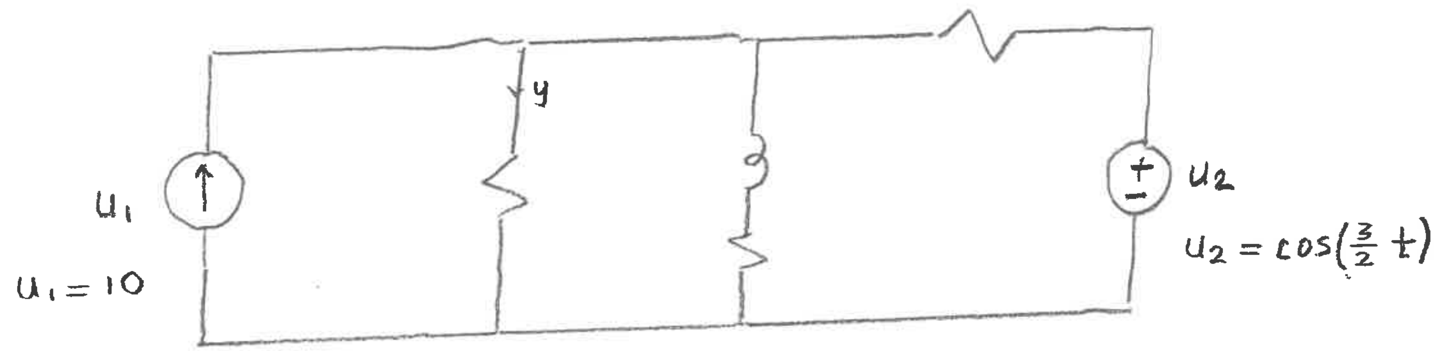
14) **<u>PLEASE DO NOT CHEAT</u> !!!**

From the desk of Dr. A.A. Rodriguez

Problem #1

Determine $H_{1,2}$, diff eq, ts, yss, y

Compute $|H_i(j\omega)|$ & $\angle H_i(j\omega)$ (all $R = L = C = 1$)

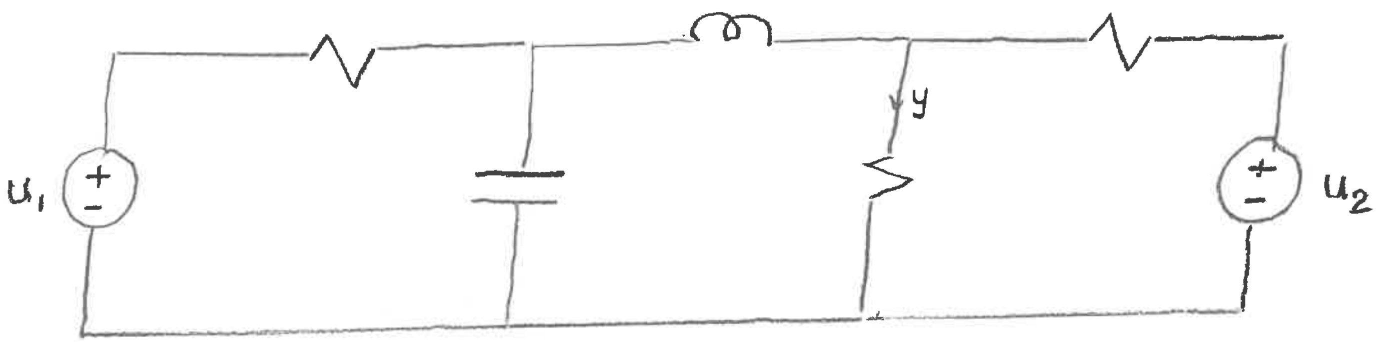


Problem # 1

Problem #2

Determine poles, t_s, $H_1(0)$, $H_2(0)$, $H_1(\infty)$, $H_2(\infty)$

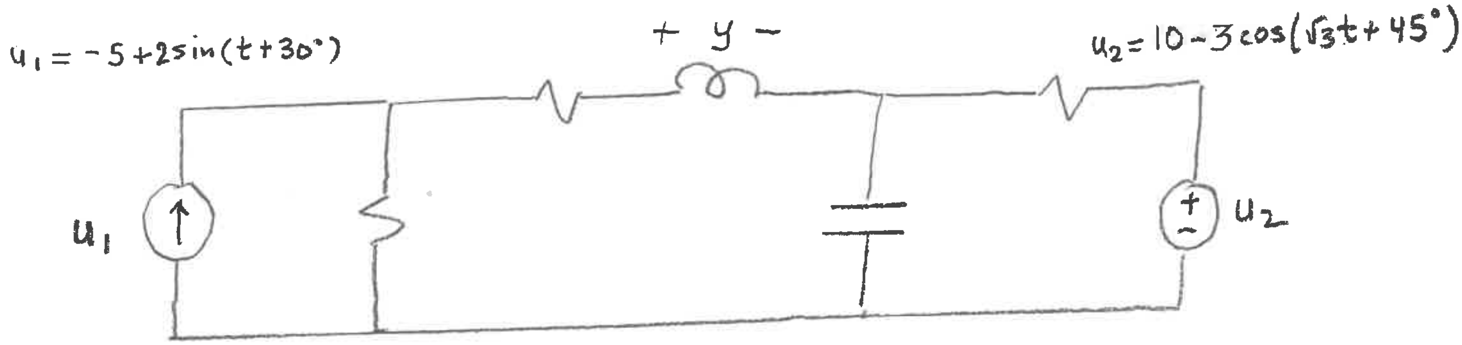
(all $R=L=C=1$)



Problem #2

Problem #3

Determine $H_{1,2}$, t_s, diffeq, y_{ss} (all $R=L=C=1$)

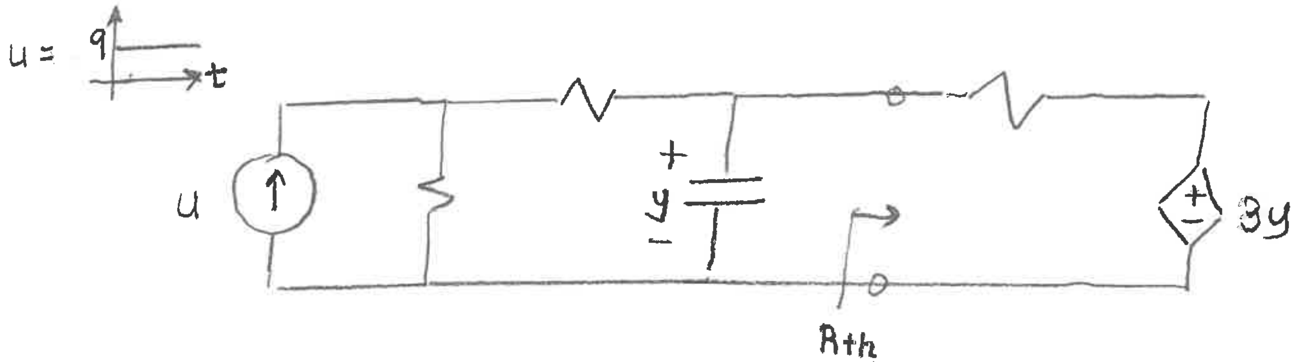


Problem # 3

Problem #4

Determine H, diff eq, y, y_{ss} (all R=L=C=1)

Hint: Find R_{th}



Problem #4

Problem #5

Determine t_s, y_{ss}, y

$$H(s) = \frac{s(s^2+1)}{(s+1)(s^2-6s+25)(s^2+2s+4)}$$

$$u(t) = 7 + 2\sin(0.01t - 45^\circ) + 3\cos(t + 30^\circ) + 4\sin(100t + 135^\circ)$$

Note: Compute all coefficients associated with constants & sinusoids in y
Show how to compute coefficients associated with instabilities in y

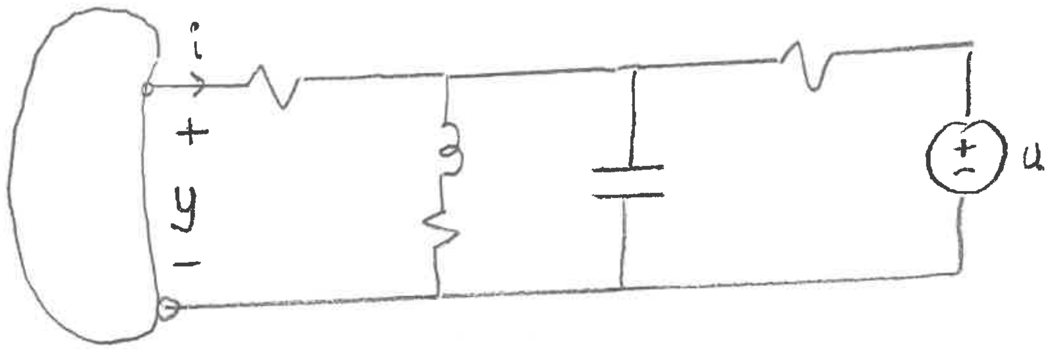
Problem #5

Problem #6

Determine an s-domain Thevenin equivalent at y

(Specify Z_{th} & V_{th})

(all $R=L=C=1$)



Problem # 6

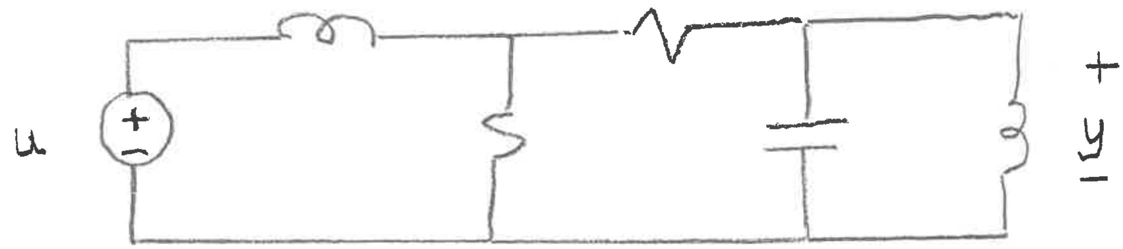
Problem # 7

Determine H, y_{ss}, t_s

Compute all coefficients in y_{ss} approximately!

Hint: poles $= \begin{Bmatrix} -0.3017 \pm j\,1.0815 \\ -0.3966 \end{Bmatrix}$

(all $R = L = C = 1$)



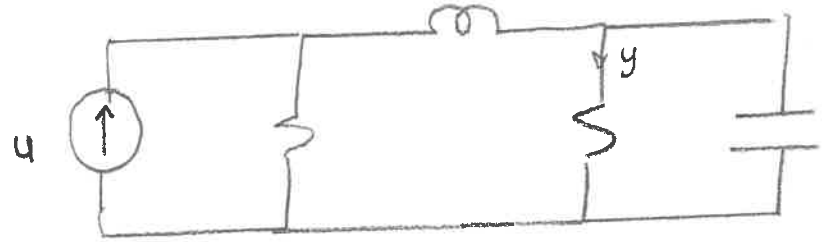
$u(t) = 6 + 100 \sin(0.01t - 45°) - 4 \cos(100t + 60°)$

Problem # 7

Problem # 8

Determine & plot $|H(j\omega)|$, $\angle H(j\omega)$

(all $R=L=C=1$)



Problem # 8

Exam #2

Fri 4-19-24

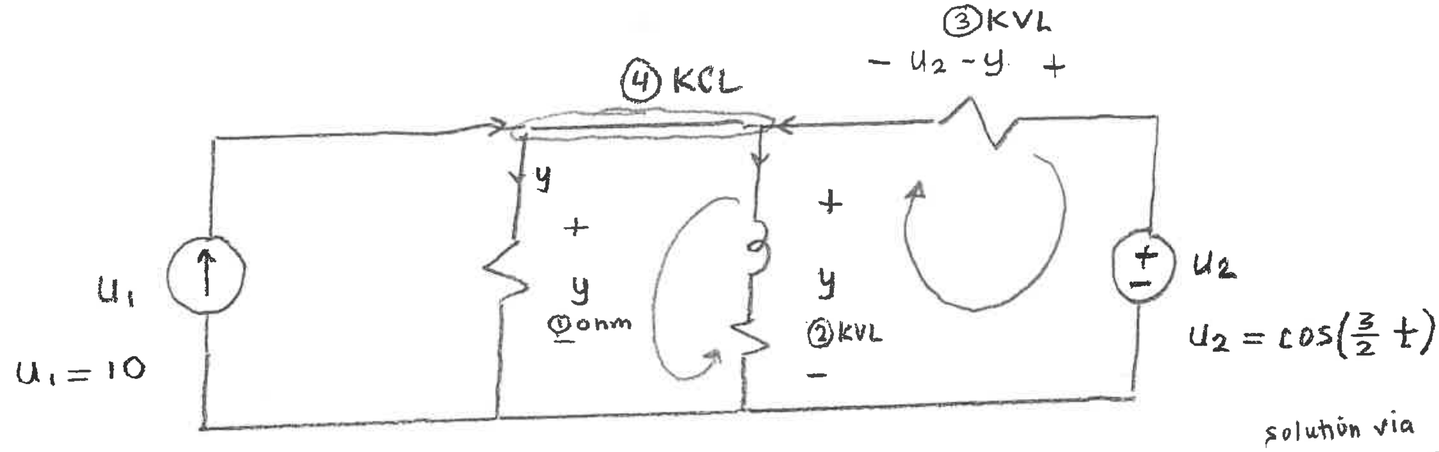
Solutions

(21 pages total)

Problem #1

Determine $H_{1,2}$, diff eq, t_s, y_{ss}, y

Compute $|H_i(j\omega)|$ & $\angle H_i(j\omega)$ (all $R=L=C=1$)



solution via Ohm, KVL, KCL

$$\boxed{u_1 + \left(\frac{u_2-y}{1}\right) = \left(\frac{y}{1}\right) + \left(\frac{y}{s+1}\right)}$$

$$\Rightarrow u_1+u_2 = y\left[1+1+\frac{1}{s+1}\right]$$

$$\Rightarrow y\left[2s+\cancel{2+1}^{\,3}\right] = (s+1)(u_1+u_2)$$

$$\Rightarrow y\,2\left[s+\frac{3}{2}\right] = (s+1)(u_1+u_2)$$

$$\Rightarrow y = \left[\frac{\frac{1}{2}(s+1)}{s+\frac{3}{2}}\right](u_1+u_2)$$

$$|H_i(j\omega)| = \frac{1}{2}\frac{\sqrt{\omega^2+1}}{\sqrt{\omega^2+\frac{9}{4}}}$$

$$\angle H_i(j\omega) = \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{3/2}\right)$$

$$H_1 = H_2 = \frac{\frac{1}{2}(s+1)}{s+3/2} \Rightarrow \dot{y}+\frac{3}{2}y = \frac{1}{2}(\dot{u}_1+u_1) + \frac{1}{2}(\dot{u}_2+u_2)$$

$$t_s = \frac{5}{|\text{Re pole}|} = \frac{5}{\frac{3}{2}} = \frac{10}{3}$$

$$H(0) = \frac{1}{3}$$

$$y_{ss} = 10\,H(0) + |H(j\tfrac{3}{2})|\cos\left(\tfrac{3}{2}t + \angle H(j\tfrac{3}{2})\right)$$

$$H(0) = \frac{1/2}{3/2} = \frac{1}{3} \qquad H(j\tfrac{3}{2}) = \frac{\frac{1}{2}(j\frac{3}{2}+1)}{j\frac{3}{2}+\frac{3}{2}}$$

$$|H(j\tfrac{3}{2})| = \frac{\frac{1}{2}\sqrt{(\frac{3}{2})^2+1}}{\frac{3}{2}\sqrt{2}}$$

$$\angle H(j\tfrac{3}{2}) = \tan^{-1}\left(\frac{3/2}{1}\right) - 45^\circ$$

$$U = \frac{10}{s} + \frac{s}{s^2+\frac{9}{4}} = \frac{10(s^2+\frac{9}{4}) + s^2}{s(s^2+\frac{9}{4})} = \frac{11s^2+\frac{90}{4}}{s(s^2+\frac{9}{4})}$$

$$Y = H\,U = \left[\frac{\frac{1}{2}(s+1)}{s+3/2}\right]\left[\frac{11s^2+\frac{90}{4}}{s(s^2+\frac{9}{4})}\right] = \frac{A}{s} + \frac{B}{s+\frac{3}{2}} + \left[\frac{C}{s-j\frac{3}{2}} + *\right]$$

$$\Rightarrow y = A + Be^{-\frac{3}{2}t} + 2|C|\cos\left(\tfrac{3}{2}t + \angle C\right)$$

$$A = \lim_{s\to 0} sY = s\,H\left[\frac{10}{s} + \frac{s}{s^2+\frac{9}{4}}\right]\Bigg|_{s=0} = 10\,H(0) = 10\left(\tfrac{1}{3}\right)$$

yss !!!

decays to zero with $t_s = \frac{10}{3}$

$$C = \lim_{s\to j\frac{3}{2}} (s-j\tfrac{3}{2})Y = (s-j\tfrac{3}{2})\,H\left[\frac{10}{s} + \frac{s}{(s-j\frac{3}{2})(s+j\frac{3}{2})}\right]\Bigg|_{s=j\frac{3}{2}}$$

Problem #1

3/21

$$C = H\left[\frac{10}{s}\left(s - j\tfrac{3}{2}\right) + \frac{s}{s+j\frac{3}{2}}\right]\Bigg|_{s=j\frac{3}{2}}$$

$$= H\left(j\tfrac{3}{2}\right)\left[0 + \frac{j\,3/2}{2\,(j\,3/2)}\right]$$

$$= \frac{H\left(j\frac{3}{2}\right)}{2}$$

$$= \tfrac{1}{2}\left|H\left(j\tfrac{3}{2}\right)\right| e^{j\angle H\left(j\frac{3}{2}\right)}$$ ← This also follows by equating

$$2|C|\cos\left(\tfrac{3}{2}t + \angle C\right) = \left|H\left(j\tfrac{3}{2}\right)\right|\cos\left(\tfrac{3}{2}t + \angle H\left(j\tfrac{3}{2}\right)\right)$$

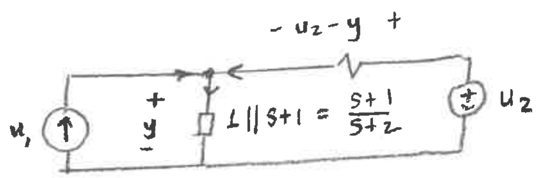
$$\Rightarrow |C| = \frac{\left|H\left(j\frac{3}{2}\right)\right|}{2} \qquad \angle C = \angle H\left(j\tfrac{3}{2}\right)$$

$$B = \lim_{s\to -\frac{3}{2}}\left(s+\tfrac{3}{2}\right)Y = \cancel{\left(s+\tfrac{3}{2}\right)}\left[\frac{\frac{1}{2}(s+1)}{\cancel{s+\frac{3}{2}}}\right]\left[\frac{11s^2+\frac{90}{4}}{s\left(s^2+\frac{9}{4}\right)}\right]\Bigg|_{s=-\frac{3}{2}}$$

$$= \tfrac{1}{2}(s+1)\left[\frac{11s^2+\frac{90}{4}}{s\left(s^2+\frac{9}{4}\right)}\right]\Bigg|_{s=-\frac{3}{2}}$$

Note: A & C are the more important coefficients because they are associated with yss!

Alternate Solution:



$$u_1 + \left(\frac{u_2 - y}{1}\right) = y\left(\frac{s+2}{s+1}\right)$$

$$(s+1)(u_1+u_2) = y\left[s+2 + s+1\right]$$

$$= y\left[2s+3\right]$$

$$= y\,2\left[s+\tfrac{3}{2}\right]$$

$$\Rightarrow y = \left[\frac{\frac{1}{2}(s+1)}{s+\frac{3}{2}}\right](u_1+u_2)$$

as we obtained above! ☺

Problem #1

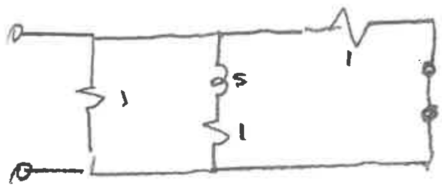
It's Always Good to Check !!!

$\boxed{4/21}$

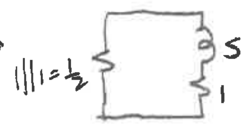
check:

I, poles & ts (set $u_1 = u_2 = 0$)

$u_1 = 0$ (open)



$u_2 = 0$ (shorted)

$\Rightarrow$ 

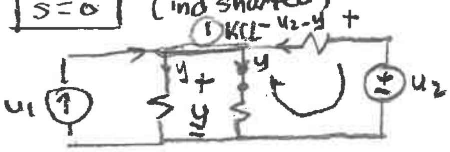
$\Rightarrow \quad Z = s + 1 + \frac{1}{2} = s + \frac{3}{2} = 0$

$\Rightarrow$ pole $= -3/2$ ✓ (as obtained earlier)

$ts \Rightarrow \dfrac{5}{|\text{Re pole}|} = \dfrac{5}{\frac{3}{2}} = \dfrac{10}{3}$ ✓ (as obtained earlier)

Gains:

$\boxed{s=0}$ (ind shorted)



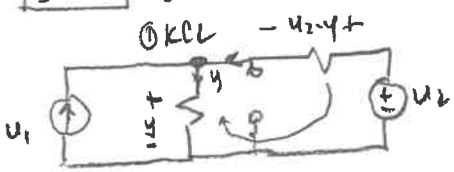
1 KCL: $u_1 + \dfrac{(u_2 - y)}{1} = y + y$

$u_1 + u_2 = 3y \quad \Rightarrow \quad y = \frac{1}{3}(u_1 + u_2)$

$H_1(0) = H_2(0) = \frac{1}{3}$ ✓

(agrees with H_i obtained earlier)

$\boxed{s=\infty}$ (ind open)



1 KCL: $u_1 + \dfrac{(u_2 - y)}{1} = y \Rightarrow 2y = u_1 + u_2$

$\Rightarrow y = \frac{1}{2}(u_1 + u_2)$

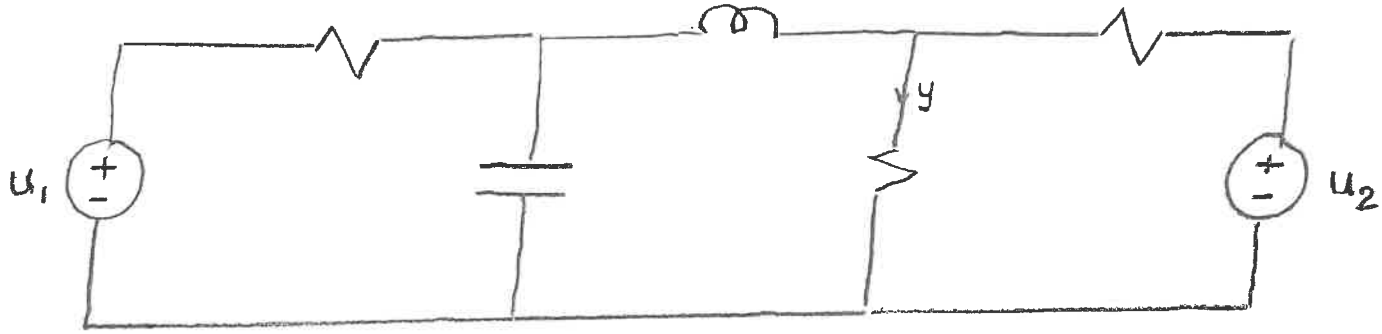
$H_1(\infty) = H_2(\infty) = \frac{1}{2}$ ✓

(agrees with H_i obtained earlier)

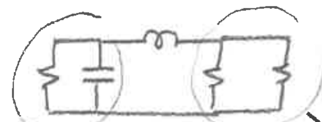
Problem #2

Determine poles, t_s, $H_1(0)$, $H_2(0)$, $H_1(\infty)$, $H_2(\infty)$

(all R=L=C=1)



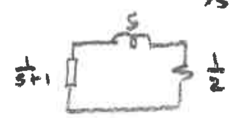
Find poles = set $u_1 = 0$ (short u_1)
$u_2 = 0$ (short u_2) $\Rightarrow$



$1 \| 1 = \frac{1}{2}$

$1 \| \frac{1}{s} = \frac{1/s}{1+1/s} = \frac{1}{s+1}$

$\Rightarrow$



$s^2 + \frac{3}{2}s + \frac{1}{2}$

$$\Rightarrow Z = \frac{1}{s+1} + s + \frac{1}{2} = \frac{1 + (s+\frac{1}{2})(s+1)}{s+1} = \frac{s^2 + \frac{3}{2}s + \frac{3}{2}}{s+1}$$

$$\Rightarrow \Phi = s^2 + \frac{3}{2}s + \frac{3}{2} = 0$$

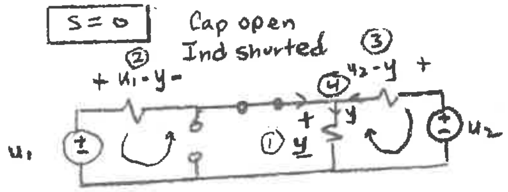
$$\Rightarrow s_{1,2} = \frac{-\frac{3}{2} \pm \sqrt{(\frac{3}{2})^2 - 4(\frac{3}{2})}}{2}$$

$$= -\frac{3}{4} \pm \sqrt{\frac{1}{4}\left(\frac{9}{4}\right) - \left(\frac{3}{2}\right)} \quad \frac{9}{16} - \frac{24}{16} = \frac{-15}{16}$$

$$= -\frac{3}{4} \pm j\frac{\sqrt{15}}{4}$$

Find ts = $\Longrightarrow t_s = \frac{5}{|\text{Re pole}|} = \frac{5}{\frac{3}{4}} = \frac{20}{3}$ sec

$\boxed{s=0}$ Cap open
Ind shorted



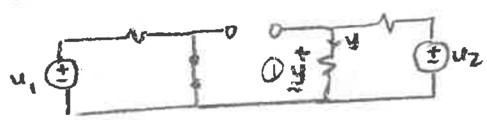
④ KCL: $\left(\frac{u_1 - y}{1}\right) + \left(\frac{u_2 - y}{1}\right) = \left(\frac{y}{1}\right)$

$\Rightarrow u_1 + u_2 = 3y$

$\Rightarrow y = \frac{1}{3}u_1 + \frac{1}{3}u_2$

$\Rightarrow H_1(0) = H_2(0) = \frac{1}{3}$

$\boxed{s=\infty}$ Cap shorted
Ind open



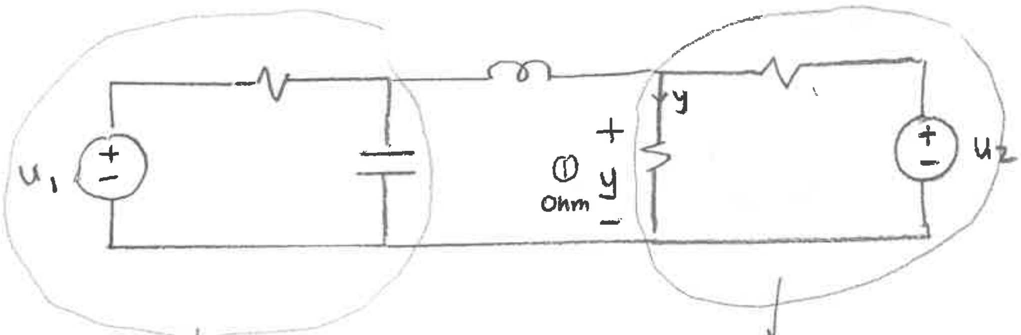
$y = 0u_1 + \frac{1}{2}u_2$ (by voltage division)

$H_1(\infty) = 0 \quad H_2(\infty) = \frac{1}{2}$

--- Now lets solve via KVL, KCL, Ohm ---
--- the long way ---

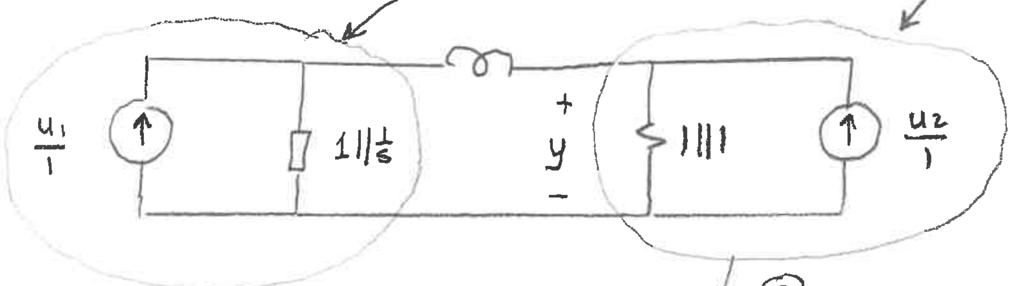
Problem #2

Solution via source transformations:



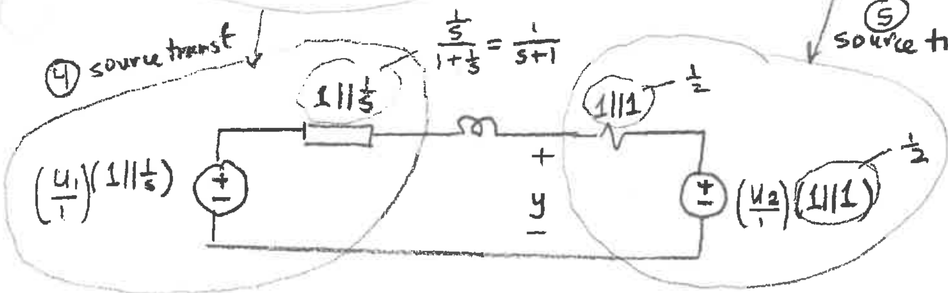
② source transf:

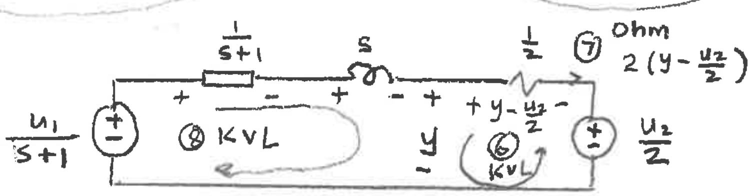
③ source transf:



④ source transf

⑤ source transf





⑧ KVL:
$$\frac{u_1}{s+1} = \left[\frac{1}{s+1} + s\right] 2\left(y - \frac{u_2}{2}\right) + y$$

$$= \left[\frac{1 + s(s+1)}{s+1}\right] 2\left(y - \frac{u_2}{2}\right) + y$$

$$u_1 = (s^2+s+1)\, 2\left(y - \frac{u_2}{2}\right) + (s+1)\, y$$

$$\frac{1}{2} u_1 + \frac{1}{2}(s^2+s+1) u_2 = y\left[s^2+s+1 + \frac{1}{2}s + \frac{1}{2}\right] = y\left[s^2 + \frac{3}{2}s + \frac{3}{2}\right]$$

$$\boxed{y = \underbrace{\left[\frac{\frac{1}{2}}{s^2 + \frac{3}{2}s + \frac{3}{2}}\right]}_{H_1} u_1 + \underbrace{\left[\frac{\frac{1}{2}(s^2+s+1)}{s^2 + \frac{3}{2}s + \frac{3}{2}}\right]}_{H_2} u_2}$$

check: same Φ & hence poles & ts!

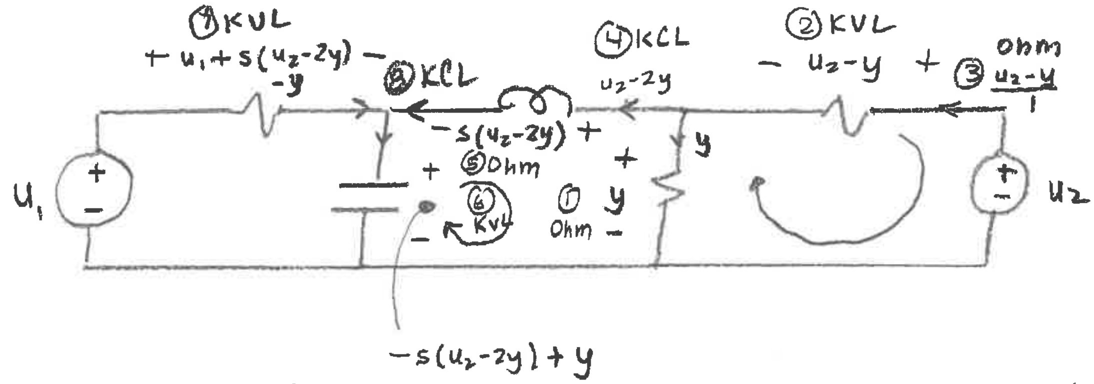
$H_1(0) = H_2(0) = \frac{1}{3}$ ✓

$H_1(\infty) = 0$ ✓ $H_2(\infty) = \frac{1}{2}$ ✓

Joy!

Problem # 2

solution via straight KVL, KCL, Ohm:



⑧ KCL:

$$\boxed{\left(\frac{u_1 + s(u_2 - 2y) - y}{1}\right) + \left(u_2 - 2y\right) = s\left[y - s(u_2 - 2y)\right]}$$

$$\Rightarrow \quad u_1 + u_2\left[s + 1 + s^2\right] = y\left[2s + 1 + 2 + s + 2s^2\right]$$

$$\Rightarrow \quad u_1 + u_2\left(s^2 + s + 1\right) = y\left[2s^2 + 3s + 3\right]$$

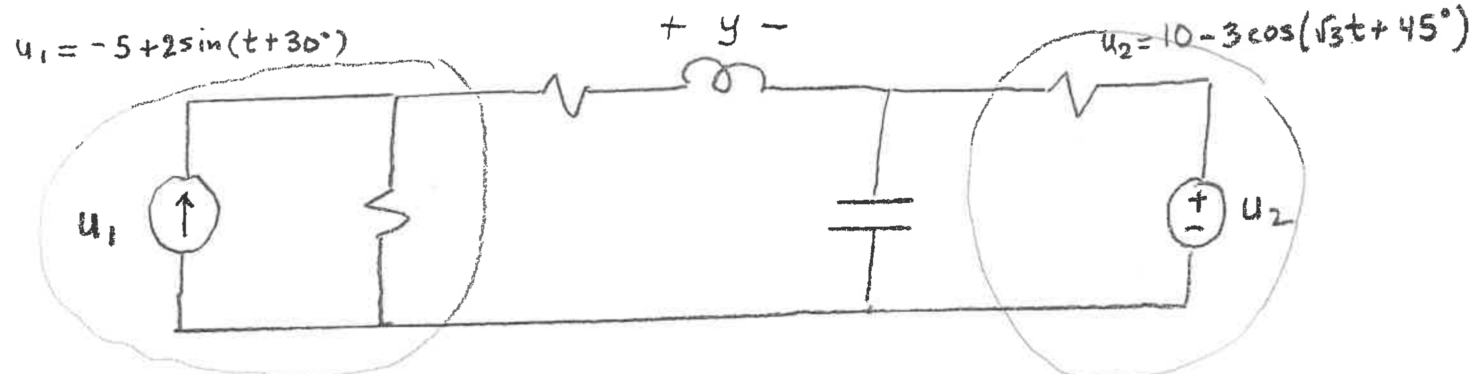
$$= y\,2\left[s^2 + \tfrac{3}{2}s + \tfrac{3}{2}\right]$$

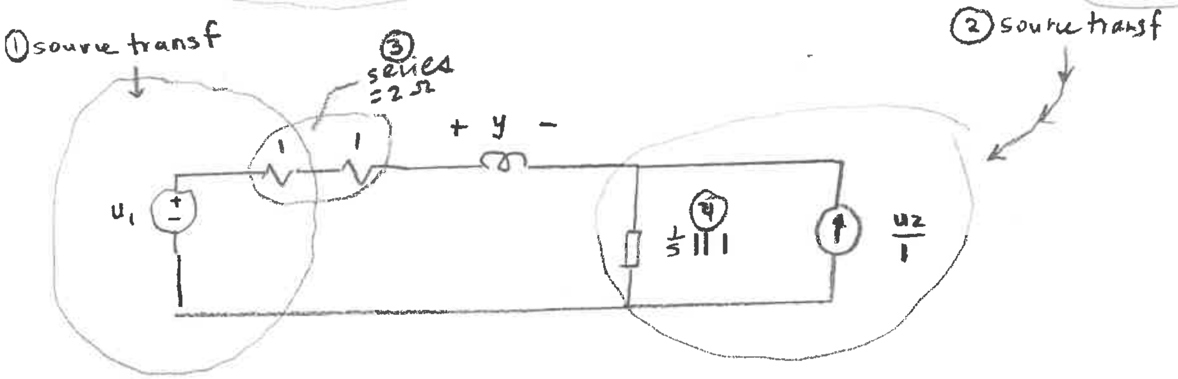
$$\Rightarrow \quad \boxed{y = \left[\frac{\frac{1}{2}}{s^2 + 3s + 3}\right]u_1 + \left[\frac{\frac{1}{2}(s^2 + s + 1)}{s^2 + \frac{3}{2}s + \frac{3}{2}}\right]u_2}$$

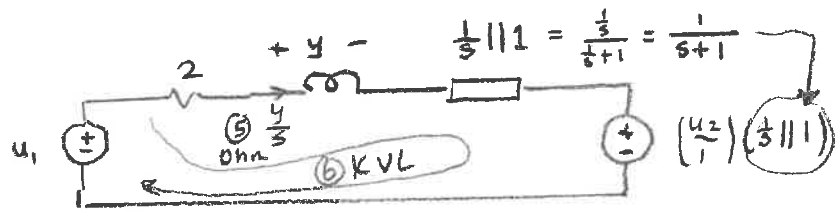
– which agrees with what we obtained earlier via source transformations!

Problem #3

Determine $H_{1,2}$, t_s, diffeq, y_{ss} (all $R = L = C = 1$)







⑥ KVL: $u_1 = 2\left(\frac{y}{s}\right) + y + \left(\frac{1}{s+1}\right)\left(\frac{y}{s}\right) + \frac{u_2}{s+1}$

$\Rightarrow y\left[\frac{2}{s} + 1 + \frac{1}{s(s+1)}\right] = u_1 - \frac{u_2}{s+1}$

$\Rightarrow y\left[2(s+1) + s(s+1) + 1\right] = s(s+1) - s\,u_2$

$\Rightarrow y\left[s^2 + 3s + 3\right] = s(s+1) - s\,u_2$

$$\Rightarrow \boxed{y = \underbrace{\left[\frac{s(s+1)}{s^2+3s+3}\right]}_{H_1} u_1 + \underbrace{\left[\frac{-s}{s^2+3s+3}\right]}_{H_2} u_2}$$

poles $= \frac{-3 \pm \sqrt{9 - 4(3)}}{2} = -\frac{3}{2} \pm j\frac{\sqrt{3}}{2}$

$\Rightarrow \ddot{y} + 3\dot{y} + 3y = \ddot{u}_1 + \dot{u}_1 - \dot{u}_2$

$\Rightarrow t_s = \frac{5}{|\text{Re poles}|} = \frac{5}{\frac{3}{2}} = \frac{10}{3}$

$y_{ss} = y_{1ss} + y_{2ss}$

$y_{1ss} = -5H_1(0) + 2|H_1(j1)|\sin(t + 30° + \angle H_1(j1))$

Method of the Transfer function (MOTF)

$H_1(0) = 0 \qquad H_1(j1) = \frac{j1(j1+1)}{3-1+j3} = \frac{(j1)(1+j1)}{2+j3} = \frac{(1e^{j90°})(\sqrt{2}e^{j45°})}{\sqrt{4+9}\;e^{j\tan^{-1}(3/2)}}$

Problem # 3

$$\Rightarrow H_1(j1) = \underbrace{\frac{(1)(\sqrt{2})}{\sqrt{4+9}}}_{|H_1(j1)|} \; e^{j\left(\overbrace{90+45^\circ - \operatorname{Tan}^{-1}(3/2)}^{\angle H_1(j1)}\right)}$$

MOTF

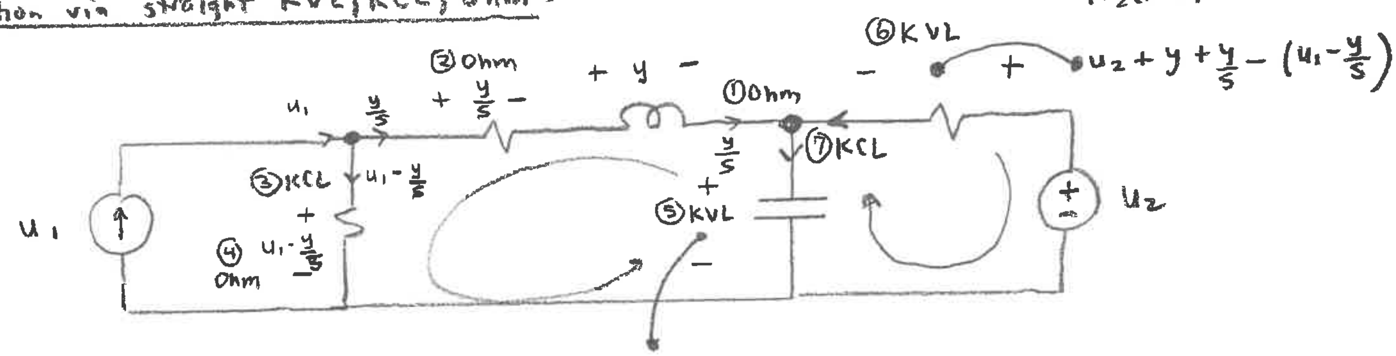
$$y_{2ss} = 10\, H_2(0) - 3\, |H_2(j\sqrt{3})| \cos(\sqrt{3}\, t + 45^\circ + \angle H_2(j\sqrt{3}))$$

$$H_2 = \frac{-s}{s^2+3s+3} \Rightarrow H_2(0) = 0$$

$$H_2(j\sqrt{3}) = \frac{-j\sqrt{3}}{-3 + j3\sqrt{3} + 3} = \frac{-j\sqrt{3}}{j3\sqrt{3}} = -\frac{1}{3} = \underbrace{\frac{1}{3}}_{|H_2(j\sqrt{3})|} e^{j\overbrace{180^\circ}^{\angle H_2(j\sqrt{3})}}$$

---

solution via straight KVL, KCL, Ohm =



$$-y - \frac{y}{s} + \left(u_1 - \frac{y}{s}\right)$$

⑦ KCL =

$$\frac{y}{s} + \left(\frac{u_2 + y + \frac{y}{s} - (u_1 - \frac{y}{s})}{1}\right) = s\left[\left(u_1 - \frac{y}{s}\right) - y - \frac{y}{s}\right]$$

$$\Rightarrow y\left[\frac{1}{s} + 1 + \frac{1}{s} + \frac{1}{s} + 1 + s + 1\right] = u_1[1+s] + u_2[-1]$$

$$\Rightarrow y\left[\frac{3}{s} + s + 3\right] = (s+1)u_1 - u_2$$

$$\Rightarrow y\left[s^2 + 3s + 3\right] = s(s+1)u_1 - s u_2$$

$$\Rightarrow y = \left[\frac{s(s+1)}{s^2+3s+3}\right]u_1 + \left[\frac{-s}{s^2+3s+3}\right]u_2$$

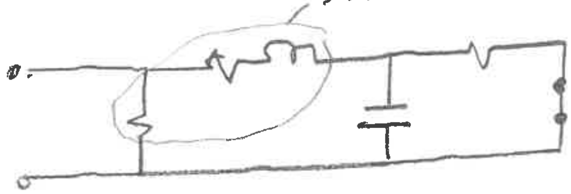
— which agrees with what we obtained earlier via source transf

$\boxed{\text{Problem \#3}}$

$\boxed{10/21}$

Check: — $\boxed{\text{It's Always Good to Check!}}$

Φ, poles & ts: set $u_1 = u_2 = 0$



$$\boxed{1 \| s+2 = \frac{s+2}{s+3} \quad = \frac{1}{s}} \Rightarrow Z = \frac{s+2}{s+3} + \frac{1}{s} = \frac{s(s+2)+s+3}{s(s+3)}$$

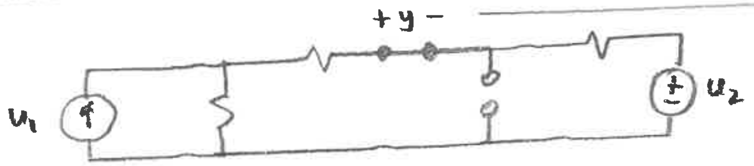
$$= \frac{s^2+3s+3}{s(s+3)}$$

$\Phi = s^2+3s+3$ as obtained earlier ✓ ✓

.. hence same poles & ts ✓

Gains:

$\boxed{s=0}$ (short ind, open cap)

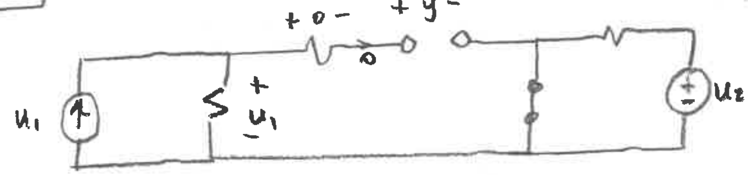


$\to y = 0\,u_1 + 0\,u_2$

$\Rightarrow H_1(0) = H_2(0) = 0$ ✓

(agrees with Hi obtained earlier)

$\boxed{s=\infty}$ (open ind, short cap)



$\to y = u_1 \Rightarrow H_1(\infty) = 1$

$H_2(\infty) = 0$ ✓

(agrees with Hi obtained earlier)

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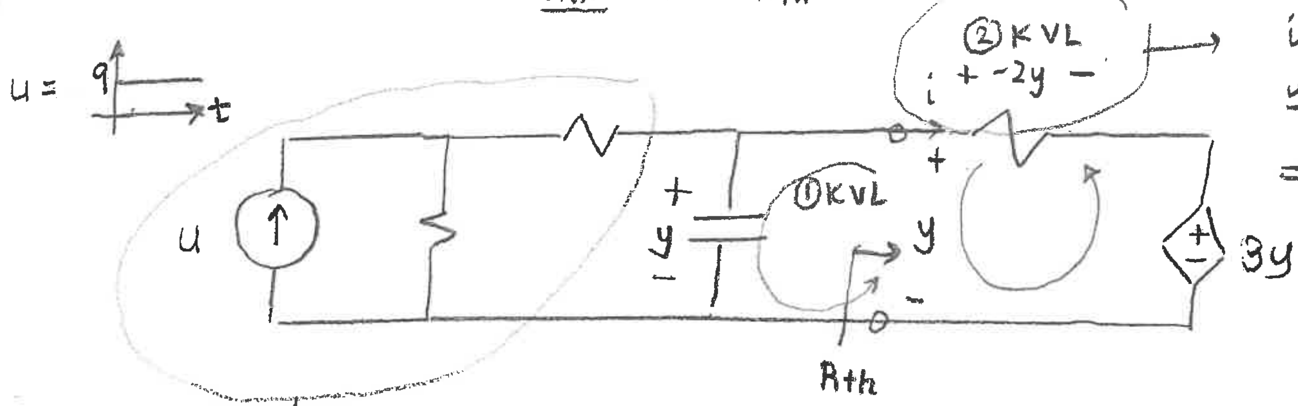
Problem #4

Determine H, diff eq, y, yss (all R=L=C=1)

Hint: Find Rth

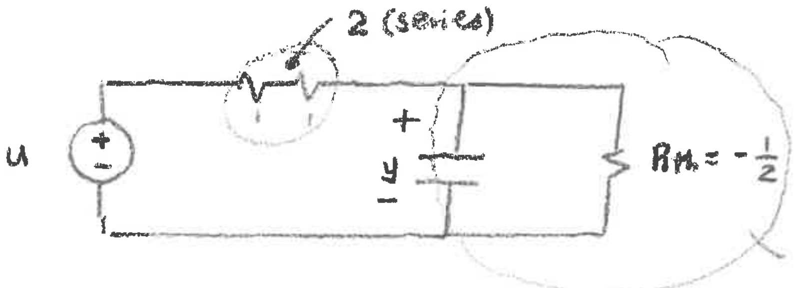


$i = -2y$

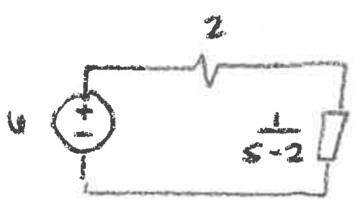
$y = -\frac{1}{2} i$

$\Rightarrow \boxed{R_{th} = -\frac{1}{2}}$

source transf:



$$\frac{1}{s} \| (-\tfrac{1}{2}) = \frac{\frac{-1/2}{s}}{\frac{1}{s} - 1/2} = \frac{-1/2}{1 - \frac{1}{2}s} = \frac{-1}{2-s} = \frac{1}{s-2}$$



voltage division

$$y = \left[\frac{\left(\frac{1}{s-2}\right)}{2 + \left(\frac{1}{s-2}\right)}\right] u = \left[\frac{1}{2s-4+1}\right] u = \left[\frac{1}{2s-3}\right] u = \left[\frac{1/2}{s - 3/2}\right] u$$

$$H = \frac{1/2}{s - 3/2} \qquad \dot{y} - \frac{3}{2} y = \frac{1}{2} u$$

$$Y = HU = \left[\frac{1/2}{s - 3/2}\right]\left(\frac{9}{s}\right) = \frac{A}{s} + \frac{B}{s - 3/2}$$

$$y = A + Be^{\frac{3}{2}t} \leftarrow y_{ss}$$

expected from MOTF!

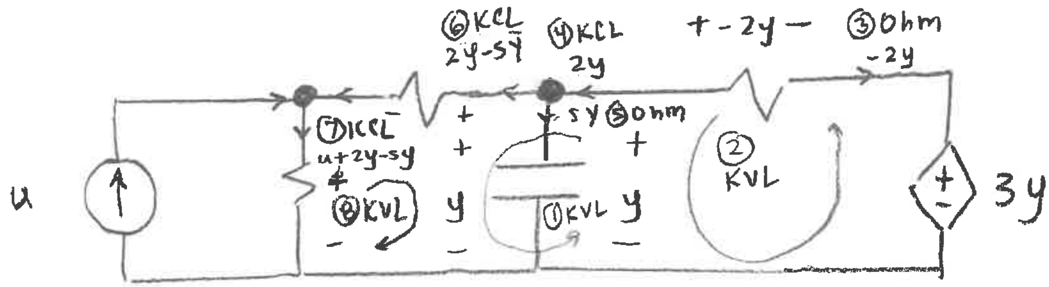
$$A = \lim_{s \to 0} sY = s\, H \frac{9}{s}\Big|_{s=0} = 9\, H(0) = 9\left(\frac{1/2}{-3/2}\right) = 9\left(-\frac{1}{3}\right) = -3$$

$$B = \lim_{s \to \frac{3}{2}} (s - \tfrac{3}{2}) Y = \cancel{(s - 3/2)} \left[\frac{1/2}{\cancel{s - \frac{3}{2}}}\right] \frac{9}{s}\Big|_{s=\frac{3}{2}} = \left(\frac{1}{2}\right)\left(\frac{9}{\frac{3}{2}}\right) = 3$$

Problem # 4

12/21

solution via straight KVL, KCL, Ohm:



$$⑧ \text{KVL}: \boxed{1(u + 2y - sy) = -[2y - sy] + y}$$

$$\Rightarrow \quad u = y[-2 + s - 2 + s + 1]$$

$$= y[2s - 3]$$

$$= y\,2\left[s - \tfrac{3}{2}\right]$$

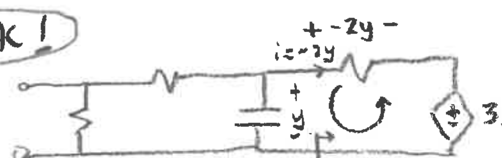
$$\Rightarrow \quad y = \underbrace{\left[\frac{1/2}{s - 3/2}\right]}_{H} u$$

--- agrees with what we got earlier via source transformations

---

check: — It's Always Good to Check!

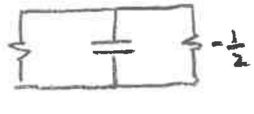
Φ, poles, ts: set $u = 0$ $\Longrightarrow$



$2y = -i$

$y = -\frac{1}{2} i \Rightarrow$ $R_{th} = -1/2$ as obtained earlier!

$\Longrightarrow$ $1+1=2$



$$\Longrightarrow -\tfrac{1}{2} \| 2 = \frac{(-\frac{1}{2})(2)}{-\frac{1}{2}+2} = \frac{-1}{\frac{3}{2}} = -\frac{2}{3}$$

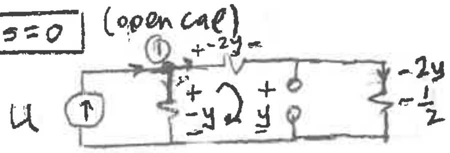
$$Z = -\tfrac{2}{3} + \tfrac{1}{s} = \frac{-\frac{2}{3}s + 1}{s}$$

$$= \frac{-\frac{2}{3}\left[s - \frac{3}{2}\right]}{s}$$

$\Phi = s - 3/2$ (as obtained above) --- hence same poles & ts ✓

Gains:

$\boxed{s = 0}$ (open cap)

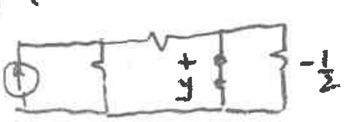


$①$ KCL: $u_1 = (-y) + (-2y) = -3y$

$y = -\frac{1}{3} u \quad H(0) = -\frac{1}{3}$ ✓

agrees with H obtained above!

$\boxed{s = \infty}$ (short cap)



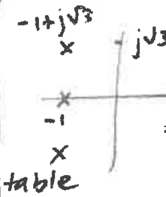
$\Rightarrow \; y = 0 \Rightarrow H(\infty) = 0$ ✓

agrees with H obtained above!

Problem #5

Determine t_s, y_{ss}, y

Note: This sinusoid will not pass through H since $H(j1)=0$



$$H(s) = \frac{s(s^2+1)}{(s+1)(s^2-6s+25)(s^2+2s+4)}$$

-1 stable $\quad 3 \pm j4$ (unstable) $\quad -1 \pm j\sqrt{3}$ stable

$$t_s = \frac{5}{|\text{Re slowest pole}|} = \frac{5}{1} = 5 \text{ sec}$$

$$u(t) = 7 + 2\sin(0.01t - 45^\circ) + 3\cos(t+30^\circ) + 4\sin(100t+135^\circ)$$

$= 2\cos(0.01t - 45^\circ - 90^\circ)$, -135°

$= 4\cos(100t + 135 - 90)$, 45°

Note: This dc term will not pass through H since $H(0)=0$.

Note: Compute all coefficients associated with constants & sinusoids in y. Show how to compute coefficients associated with instabilities in y

Recall:

$$M \longleftrightarrow \frac{M}{s}$$

$$|M|\cos(w_0 t + \angle M) \longleftrightarrow \frac{(|M|\cos\angle M)s + w_0|M|\sin\angle M}{s^2+w_0^2}$$

dc & sinusoidal components of y_{ss}

$$y_{ss} = \underbrace{|B|e^{3t}\cos(4t+\angle B)}_{\text{dominant growing (unstable) exponential sinusoid}} + 7H(0) + 2|H(j0.01)|\cos(0.01t - 135^\circ + \angle H(j0.01)) + 3|H(j1)|\cos(t+30^\circ+\angle H(j1)) + 4|H(j100)|\cos(100t + 45^\circ + \angle H(j100))$$

$-45^\circ - 90^\circ$

$135 - 90^\circ$

zero since $H(0)=0$

zero since $H(j1)=0$

from partial fraction expansion below!

from HOTF!

$$U = \frac{7}{s} + \left(\frac{2\cos(-135^\circ)s + 0.01\, 2\sin(-135^\circ)}{s^2+10^{-4}}\right) + \left(\frac{(3\cos 30^\circ)s + (1)3\sin 30^\circ}{s^2+1}\right) + \left(\frac{4\cos 45^\circ s + 100(4\sin 45^\circ)}{s^2+10^4}\right)$$

$$= \frac{(\text{some numerator})}{s(s^2+10^{-4})(s^2+1)(s^2+10^4)}$$

← denominator is what is critical for forthcoming partial fraction expansion

$$Y = HU = \left[\frac{\cancel{s(s^2+1)}}{(s+1)(s^2-6s+25)(s^2+2s+4)}\right]\left[\frac{\text{some numerator}}{\cancel{s}(s^2+10^{-4})\cancel{(s^2+1)}(s^2+10^4)}\right]$$

$-1 \quad 3+j4 \quad -1+j\sqrt{3} \quad j0.01 \quad j100$

Note: Cancellation of s's & (s^2+1)'s is expected since $H(0)=0$ & $H(j1)=0$

$$= \frac{A}{s+1} + \left[\frac{B}{s-3-j4} + *\right] + \left[\frac{C}{s+1-j\sqrt{3}} + *\right] + \left[\frac{D}{s-j0.01} + *\right] + \left[\frac{E}{s-j100} + *\right]$$

$$y(t) = Ae^{-t} + 2|B|e^{3t}\cos(4t+\angle B) + 2|C|e^{-t}\cos(\sqrt{3}t+\angle C) + 2|D|\cos(0.01t+\angle D) + 2|E|\cos(100t+\angle E)$$

Ae^{-t}: decaying stable exp

$2|B|e^{3t}\cos(4t+\angle B)$: growing unstable exp sinusoid

$2|C|e^{-t}\cos(\sqrt{3}t+\angle C)$: decaying stable exp sinusoid

These are just $2|H(j0.01)|\cos(0.01t - 135^\circ + \angle H(j0.01)) + 4|H(j100)|\cos(100t + 45^\circ + \angle H(j100))$ ← from HOTF above

Problem # 5

Lets compute the important coefficients :

$$B = \lim_{s \to 3+j4} (s-3-j4)\,Y = \left[\frac{1}{(s+1)(s-3+j4)(s^2+2s+4)}\right]\left[\frac{\text{some numerator}}{(s^2+10^{-4})(s^2+10^4)}\right]\Bigg|_{s=3+j4}$$

- to calculate B one would need to find the "some numerator" & then plug in $s=3+j4$ above to get B
--- can use MATLAB!

From ⍟, it follows that

$$D = \frac{2|H(j0.01)|}{2}\, e^{j\left[-135^\circ + \angle H(j0.01)\right]}$$

$$E = \frac{4|H(j100)|}{2}\, e^{j\left[45^\circ + \angle H(j100)\right]}$$

Note: The above can also be obtained from the following standard partial fraction expansion coefficient formulae for D & E:

$$D = \lim_{s \to j0.01} (s - j0.01)\,Y(s)$$

$$E = \lim_{s \to j100} (s - j100)\,Y(s)$$

A & C are less important because they are associated with decaying (stable) exponentials --- but they can be computed using the following formulae for A & C:

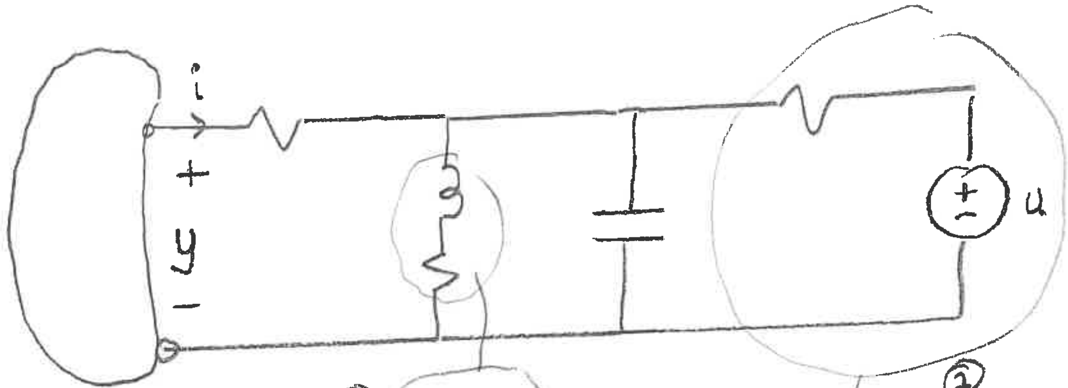
$$A = \lim_{s \to -1} (s+1)\,Y(s)$$

$$C = \lim_{s \to -1+j\sqrt{3}} (s+1-j\sqrt{3})\,Y(s)$$

Problem #6

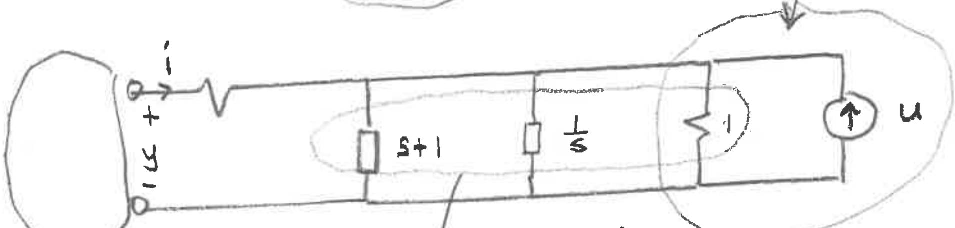
Determine an s-domain Thevenin equivalent at y

(Specify Z_{th} & V_{th}) (all R=L=C=1)



① series: $s+1$

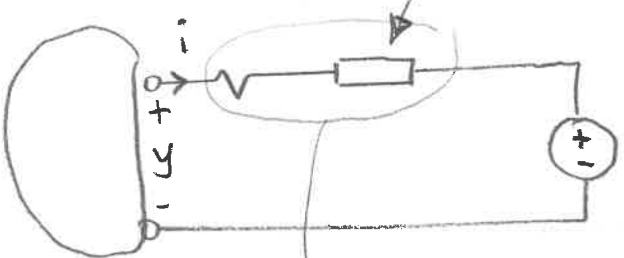
② source transf



③ parallel

$(s+1)\,||\,(\tfrac{1}{s})\,||\,(1)$

④ source transf



$$\boxed{V_{th} = u \times \left[(s+1)\,||\,(\tfrac{1}{s})\,||\,(1)\right]}$$

⑤ series

$$\boxed{Z_{th} = 1 + \left[(s+1)\,||\,(\tfrac{1}{s})\,||\,(1)\right]}$$

Best way to write V_{th} & Z_{th} !!!

<u>Note:</u> $(s+1)\,||\,(\tfrac{1}{s})\,||\,(1) = (s+1)\,||\left[\dfrac{(\frac{1}{s})(1)}{\frac{1}{s}+1}\right]$

$= (s+1)\,||\left[\dfrac{1}{s+1}\right]$

$= \dfrac{\cancel{(s+1)}\left(\frac{1}{\cancel{s+1}}\right)}{s+1+\frac{1}{s+1}}$

$= \dfrac{s+1}{(s+1)^2+1} = \dfrac{s+1}{s^2+2s+2}$

This is like a check!!!

<u>Note:</u>

At dc ($s=\infty$), $Z_{th} = 1 + [(0+1)||\infty||1]$

$= 1 + [1||1]$

$= 1 + \tfrac{1}{2}$

$= 3/2$

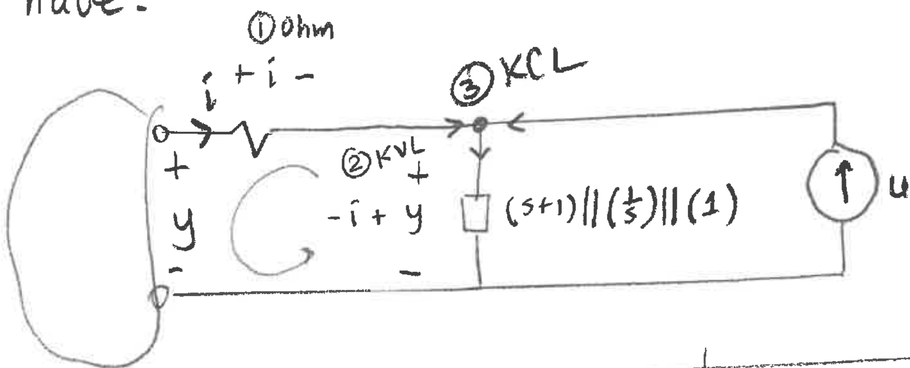
For $s=\infty$, $Z_{th} = 1 + [\infty||0||1]$

$= 1 + 0$

$= 1$

Problem # 6

From our 2nd cct #; it follows that we have:



③ KCL =

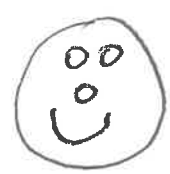
$$\boxed{\overrightarrow{i} + \overleftarrow{u} = \frac{-i+y}{(s+1)\,||\,(\frac{1}{s})\,||\,(1)}}$$

$$\Longrightarrow \quad (i+u)\left[(s+1)\,||\,(\tfrac{1}{s})\,||\,(1)\right] = -i + y$$

$$\Longrightarrow \quad \boxed{y = \left[1 + (s+1)\,||\,(\tfrac{1}{s})\,||\,(1)\right] i + \left[(s+1)\,||\,(\tfrac{1}{s})\,||\,(1)\right] u}$$

$$\boxed{Z_{th} = 1 + (s+1)\,||\,(\tfrac{1}{s})\,||\,(1)} \qquad \boxed{V_{th} = \left[(s+1)\,||\,(\tfrac{1}{s})\,||\,(1)\right] u}$$

– as we got earlier via source transformations...



Problem # 7

Determine H, y_{ss}, t_s

Compute all coefficients in y_{ss} approximately!

Hint: poles $= \left\{ \begin{array}{l} -0.3017 \pm j\,1.0815 \\ -0.3966 \end{array} \right\}$

(all R = L = C = 1)

all stable

$t_s = \dfrac{5}{|\text{Re slowest pole}|} = \dfrac{5}{0.3017}$

Note: we can use MOTF to write expression for y_{ss} before analyzing circuit

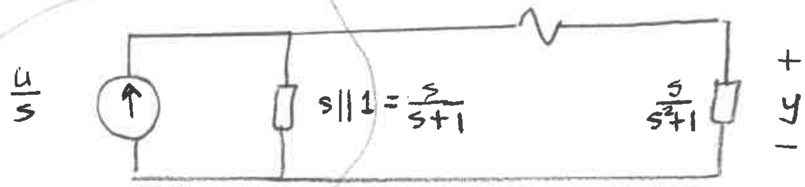
$u(t) = 6 + 100\sin(0.01t - 45°) - 4\cos(100t + 60°)$

$y_{ss} = 6\,H(0) + 100\,|H(j0.01)|\sin(0.01t - 45° + \angle H(j0.01)) - 4\,|H(j100)|\cos(100t + 60° + \angle H(j100))$

from MOTF! (stable system)

① source transf

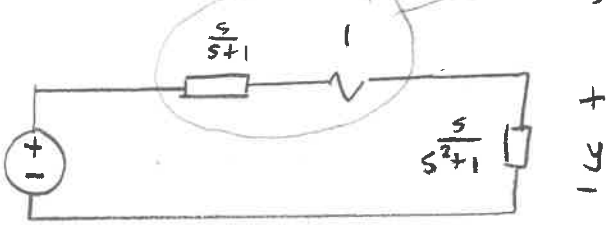
② parallel: $\frac{1}{s} \| s = \dfrac{\left(\frac{1}{s}\right)(s)}{\frac{1}{s} + s} = \dfrac{1}{\frac{1}{s} + s} = \dfrac{s}{s^2+1}$



③ source transf

④ series: $1 + \dfrac{s}{s+1} = \dfrac{s+1+s}{s+1} = \dfrac{2s+1}{s+1}$

$\left(\dfrac{u}{s}\right)\left(\dfrac{s}{s+1}\right) = \dfrac{u}{s+1}$



By voltage division: $\Longrightarrow$

$$y = \left[\frac{\frac{s}{s^2+1}}{\frac{2s+1}{s+1} + \frac{s}{s^2+1}} \right] \frac{u}{s+1}$$

multip top & bottom by $(s+1)(s^2+1)$:

$$= \left[\frac{s\,\cancel{(s+1)}}{(2s+1)(s^2+1) + s(s+1)} \right] \frac{u}{\cancel{s+1}}$$

$$= \left[\frac{\frac{1}{2}\,s}{s^3 + s^2 + \frac{3}{2}s + \frac{1}{2}} \right] u$$

H

$$\begin{array}{r} s^2 + 1 \\ 2s + 1 \\ \hline 2s^3 + 2s \\ s^2 + 1 \\ \hline 2s^3 + s^2 + 2s + 1 \\ + \quad s^2 + s \\ \hline 2s^3 + 2s^2 + 3s + 1 \end{array}$$

$= 2\left[s^3 + s^2 + \frac{3}{2}s + \frac{1}{2}\right]$

Problem #7

MOTF (stable system)

$$y_{ss} = 6H(0) + 100\,|H(j0.01)|\,\sin(0.01t - 45^\circ + \angle H(j0.01))$$

$$- 4\,|H(j100)|\,\cos(100t + 60^\circ + \angle H(j100))$$

0.01 $\quad 90^\circ$ $\quad \sim \frac{1}{2}10^{-4}$ $\quad -180^\circ$

$$H(s) = \left[\frac{\frac{1}{2}s}{s^3 + s^2 + \frac{3}{2}s + \frac{1}{2}}\right]$$

$H(0) = 0$ — yss will have no constant (dc) component in it!

H looks like a differentiator at low freq's

$$H(s) \overset{s\ \text{small}}{\approx} \frac{\frac{1}{2}s}{\frac{1}{2}} = s \implies H(j0.01) \cong j0.01 = 0.01e^{j90^\circ}$$

$|H(j0.01)| \cong 0.01$

$\angle H(j0.01) \cong 90^\circ$

$$H(s) \overset{s\ \text{large}}{\approx} \frac{\frac{1}{2}s}{s^3} = \frac{1/2}{s^2} \implies H(j100) \cong \frac{1/2}{(j100)^2} = \frac{1/2}{(10^2 e^{j90^\circ})^2}$$

$$= \frac{1/2}{10^4 e^{j180^\circ}}$$

$$= \left(\tfrac{1}{2}\right)10^{-4} e^{-j180^\circ}$$

H looks like a double integrator at high freq's

$|H(j100)| \cong \frac{1}{2}10^{-4}$

$\angle H(j100) \cong -180^\circ$

Problem #7

It's Always Good to Check!

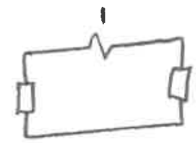
Check:

Φ, poles, ts: Set $u=0$ $\Rightarrow$ $u=0$ (short)

$s \| 1 = \frac{s}{s+1}$

$\frac{1}{s} \| s = \frac{(\frac{1}{s})s}{\frac{1}{s}+s} = \frac{1}{\frac{1}{s}+s} = \frac{s}{s^2+1}$

$\Rightarrow$



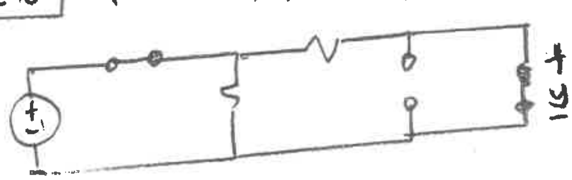
$$Z = \frac{s}{s+1} + 1 + \frac{s}{s^2+1}$$
$$= \frac{s(s^2+1) + (s+1)(s^2+1) + s(s+1)}{(s+1)(s^2+1)}$$
$$= \frac{s^3+s + s^3+s+s^2+1 + s^2+s}{(\ \ )(\ \ )}$$
$$= \frac{2s^3 + 2s^2 + 3s + 1}{(\ \ )(\ \ )}$$
$$= \frac{2\left[s^3 + s^2 + \frac{3}{2}s + \frac{1}{2}\right]}{(\ \ )(\ \ )}$$

$\Phi = s^3 + s^2 + \frac{3}{2}s + \frac{1}{2}$ as obtained earlier! ✓

$\Rightarrow$ hence same poles & ts ✓

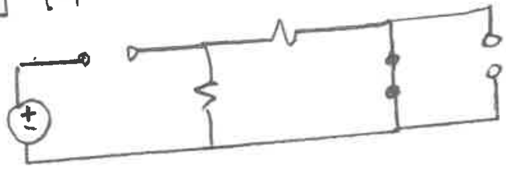
Gains:

$\boxed{s=0}$ (short inds, open cap)



$\Rightarrow$ $y=0$ $\Rightarrow$ $H(0)=0$ (agrees with H obtained earlier) ✓

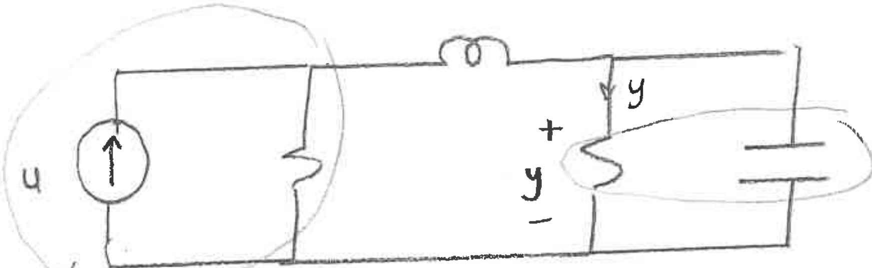
$\boxed{s=\infty}$ (open inds, short cap)



$\Rightarrow$ $y=0$ $\Rightarrow$ $H(\infty)=0$ (agrees with H obtained earlier) ✓

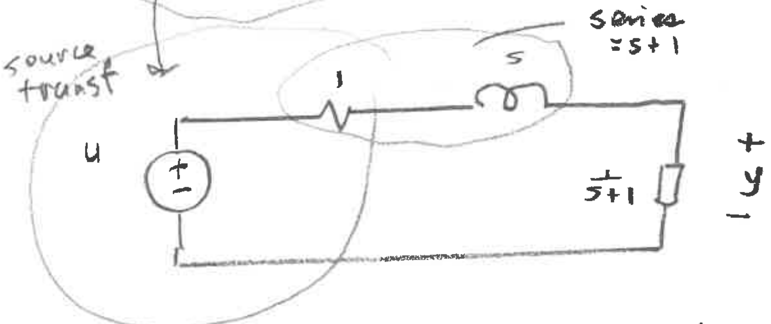
Problem # 8 $\qquad$ Determine & plot $|H(j\omega)|$, $\underline{/H(j\omega)}$ $\qquad$ 20/21

(all $R=L=C=1$)



$1 \| \frac{1}{s} = \frac{(1)(\frac{1}{s})}{1+\frac{1}{s}} = \frac{1}{s+1}$

source transf



$\Rightarrow$ By voltage division: $H = \frac{y}{u} = \frac{\left(\frac{1}{s+1}\right)}{s+1+\frac{1}{s+1}}$

$= \frac{1}{(s+1)^2+1}$

$\boxed{H(s) = \frac{1}{s^2+2s+2}}$

$H(j\omega) = \frac{1}{2-\omega^2+j2\omega}$

$|H(j\omega)| = \frac{1}{\sqrt{(2-\omega^2)^2+4\omega^2}}$

Note: This is what I use to plot $|H(j\omega)|$ & $\underline{/H(j\omega)}$

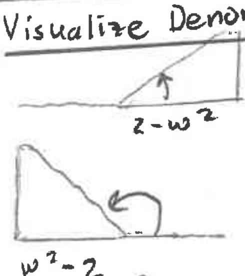
$H(0) = \frac{1}{2} = \frac{1}{2}e^{j0°}$

$H(j\sqrt{2}) = \frac{1}{2j\sqrt{2}} = \frac{1}{2\sqrt{2}}e^{-j90°}$

$H(s) \overset{s\ \text{large}}{\approx} \frac{1}{s^2}$

$|H(j\infty)| = 0 \qquad \underline{/H(j\infty)} = -180°$

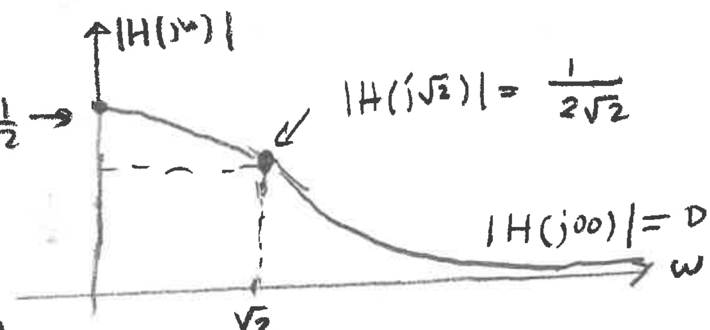
Visualize Denominator:



$\omega \in [0, \sqrt{2}]$

$\omega \in [\sqrt{2}, \infty)$

$H(0) = \frac{1}{2}$

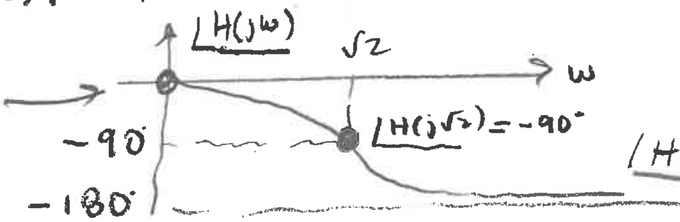


$|H(j\sqrt{2})| = \frac{1}{2\sqrt{2}}$

$|H(j\infty)| = 0$

$$\underline{/H(j\omega)} = \begin{cases} -\tan^{-1}\left(\frac{2\omega}{2-\omega^2}\right) & \omega \in [0, \sqrt{2}] \\ -\left[180° - \tan^{-1}\left(\frac{2\omega}{\omega^2-2}\right)\right] & \omega \in [\sqrt{2}, \infty) \end{cases}$$

$\underline{/H(j0)} = 0$



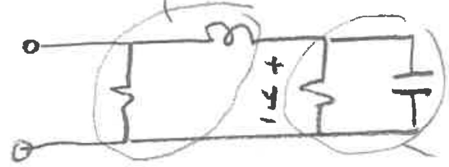
$\underline{/H(j\sqrt{2})} = -90°$

$\underline{/H(j\infty)} = -180$

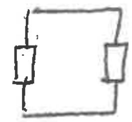
Problem # 8

Check: It's Always Good to Check!

Φ, poles, ts: set $u=0$ $\Rightarrow$



$1 \| \frac{1}{s} = \frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{1}{s+1}$



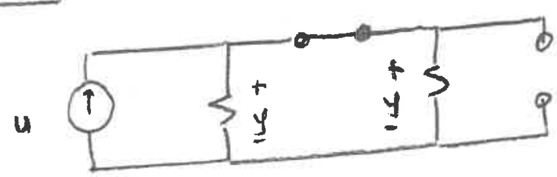
$$Z = s+1+\frac{1}{s+1} = \frac{(s+1)^2+1}{s+1} = \frac{s^2+2s+2}{s+1}$$

$\Rightarrow \Phi = s^2+2s+2$ ✓ (agrees with what was obtained earlier!)

$\Rightarrow$ hence same poles, ts! ✓

Gains:

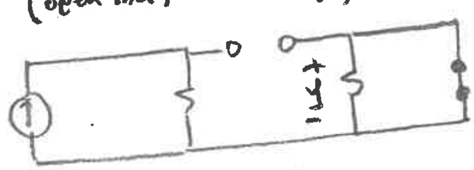
$\boxed{s=0}$ (short ind, open cap)



$\xRightarrow{KCL}$ $u = 2y \Rightarrow y = \frac{1}{2}u \Rightarrow H(0) = \frac{1}{2}$

agrees with H obtained earlier!

$\boxed{s=\infty}$ (open ind, short cap)



$\xRightarrow{KVL}$ $y = 0 \Rightarrow H(\infty) = 0$

agrees with H obtained earlier!

EEE202

Final Exam

Fri 4-26-24

8 Problems (5+3)

(17 pages follow)

EXAM RULES

1) One 8.5" x 11" sheet permitted;
   otherwise closed books, closed notes, open minds!

2) Scientific calculator is permitted; no computers and no phone!

3) **<u>NO PHONES</u> !!!**
   Phones should not be visible at all!
   A visible phone will result in a zero grade for the exam!

4) Write FULL NAME legibly on each page provided.

5) **<u>PLEASE SHOW ALL WORK</u> !!!**
   This is essential to receive partial credit!

6) Please do not submit multiple answers. You must pick an answer!!

7) Write your solutions on the sheets provided.
   No other paper/sheets/pages should be used!

8) Clearly label voltages and currents on the circuits provided.

9) Use the variables provided!
   No additional variables should be used!

10) Unreadable work will receive NO CREDIT.

11) Please place important equations and answers within boxes as we have done in lecture.

12) Please be careful with your algebra, signs, etc.

13) Please turn in your solutions to me at the end of the period!

14) **<u>PLEASE DO NOT CHEAT</u> !!!**

From the desk of Dr. A.A. Rodriguez

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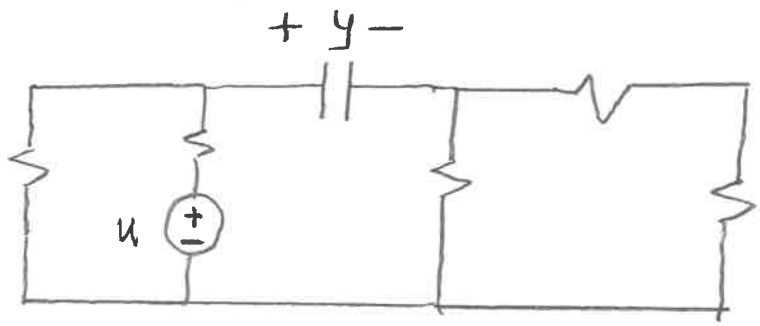
Problem #1

Determine $H_{1,2}$, diffeq, t_s, y_{ss}, y_e

Compute $|H_i(j\omega)|$ & $\angle H_i(j\omega)$ $(i=1,2)$,

(all R = L = C = 1)



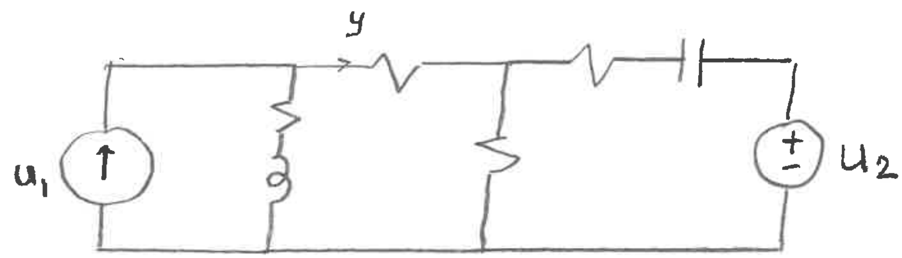
$u = 10 - 7\cos\left(\frac{6}{7}t + 45°\right)$

Problem # 1

Problem # 2

Determine poles, t_s, $H_1(0)$, $H_2(0)$, $H_1(\infty)$, $H_2(\infty)$.

(all $R = L = C = 1$)



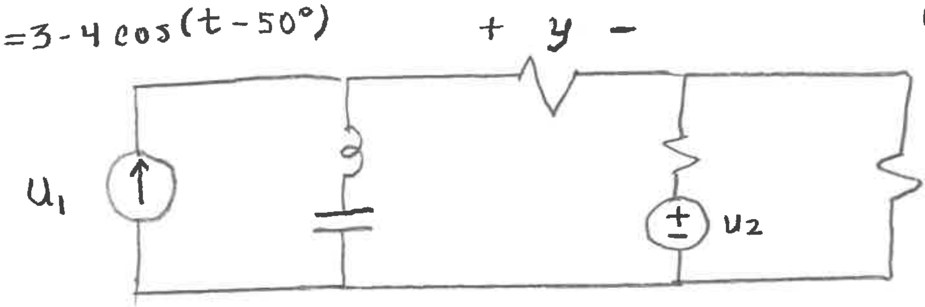
Problem #2

Problem #3

Determine $H_{1,2}$, t_s, diff eq, y_{ss} (all $R=L=C=1$)

$u_1 = 3 - 4\cos(t - 50°)$

$u_2 = -8 - 2\sin(t + 30°) + 200\cos(100t + 95°)$

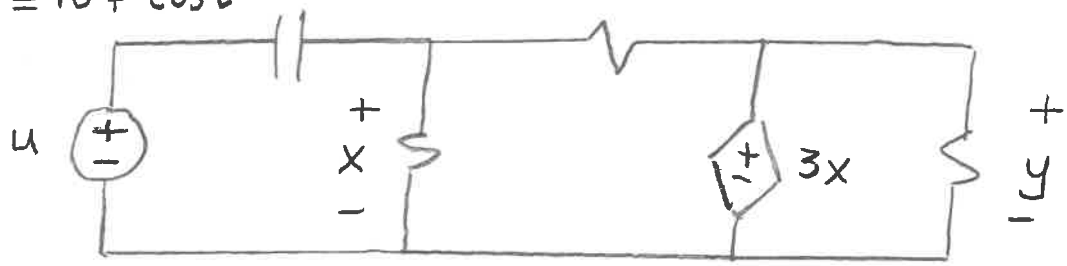


Problem # 3

Problem # 4

Determine H, diff eq, y, y_{ss} (all $R=L=C=1$)

$u(t) = 10 + \cos t$



Problem # 4

Problem # 5

Determine t_s, y_{ss}, y

$$H(s) = \frac{(s+0.01)(s^2+s+1)}{(s+2)(s^2+8s+25)(s^2-2s+4)}$$

$$u = -5 - 6\cos(0.01t - 45^\circ) + 7\sin(100t + 60^\circ)$$

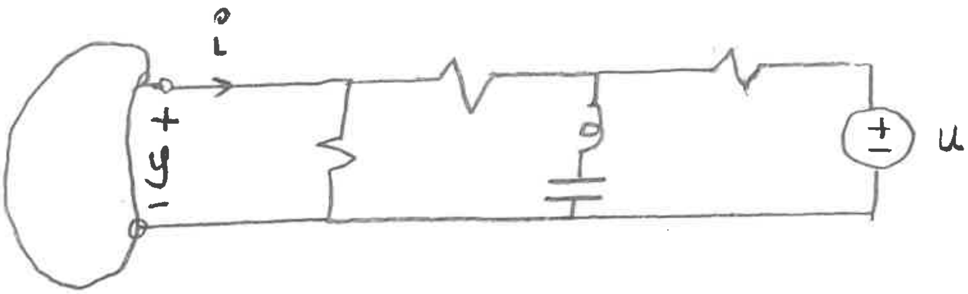
Note: compute all coefficients associated with constants & sinusoids in y.
Show how to compute coefficients associated with instabilities in y.

Problem # 5

Problem #6

Determine an s-domain Thevenin equivalent at y

(Specify Z_{th} & V_{th}!) (all $R=L=C=1$)



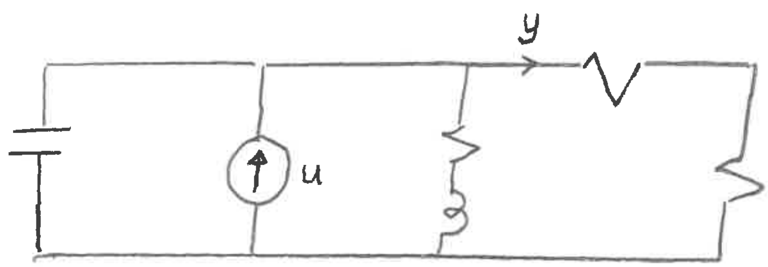
Problem # 6

Problem # 7

Determine H, y_{ss}, t_s

Compute all coefficients in y_{ss} approximately!

(all $R = L = C = 1$)



$$u = -7 - 8\cos\left(\sqrt{\tfrac{3}{2}}\,t - 30^\circ\right) - 9\sin\left(100t + 53^\circ\right)$$

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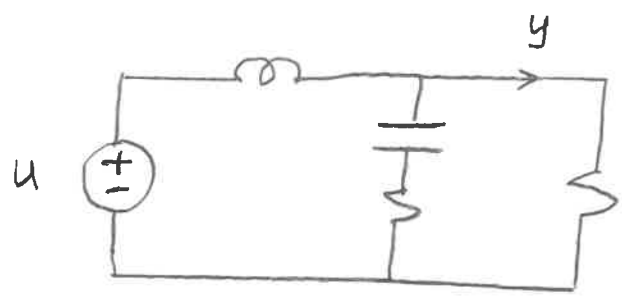
Problem # 7

Problem # 8

Determine & plot $|H(j\omega)|$ & $\angle H(j\omega)$

(all $R = L = C = 1$)



Problem # 8

EEE 202

Final Exam

Fri 4-26-24

Solutions

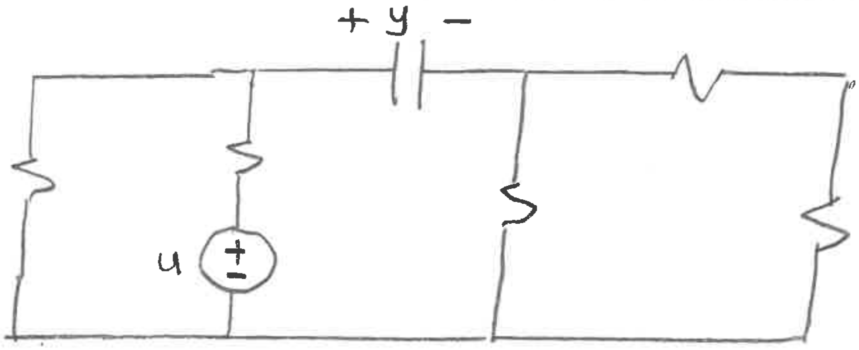
(pages 10-280)
follow

(28 pages follow)

Problem # 1

Determine $H_{1,2}$, diff eq, t_s, y_{ss}, y.

Compute $|H_i(j\omega)|$ & $\angle H_i(j\omega)$ $(i=1,2)$.



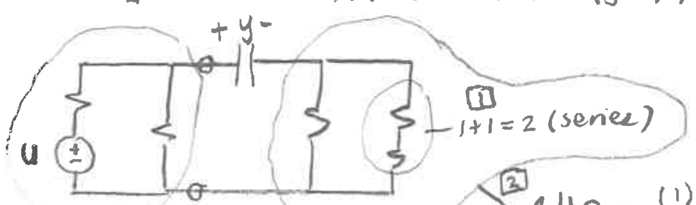
(all R = L = C = 1)

$u = 10 - 7\cos\left(\frac{6}{7}t + 45°\right)$

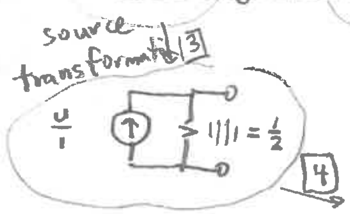
solution:

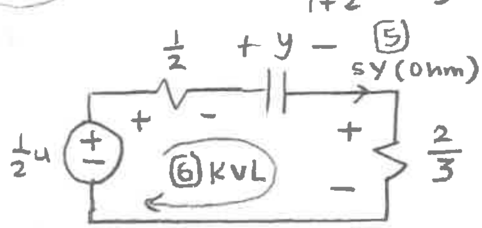
(Note: Above circuit will be stable since it only has R's, C & u)

$$y_{ss} \overset{\text{MOTF}}{=} 10\,H(0) - 7\,\left|H\left(j\tfrac{6}{7}\right)\right| \cos\left(\tfrac{6}{7}t + 45° + \angle H\left(j\tfrac{6}{7}\right)\right)$$



$1 \| 2 = \frac{(1)(2)}{1+2} = \frac{2}{3}$ (parallel)





6 KVL: $\frac{1}{2}u = \frac{1}{2}sy + y + \frac{2}{3}sy$

$= y\left[\left(\frac{1}{2}\right)s + 1 + \left(\frac{2}{3}\right)s\right]$ ($3/6$, $4/6$)

$= y\left[\frac{7}{6}s + 1\right]$

$= y\,\frac{7}{6}\left[s + \frac{6}{7}\right]$

$$\boxed{y = \left[\frac{\frac{3}{7}}{s + \frac{6}{7}}\right]u}$$

$\llcorner H$ $\Longrightarrow$ $\dot{y} + \frac{6}{7}y = \frac{3}{7}u$

$H(0) = \frac{3}{6} = \frac{1}{2}$

$H\left(j\frac{6}{7}\right) = \frac{3/7}{\frac{6}{7}(1 + j1)} = \frac{1/2}{\sqrt{2}\,e^{j45}}$

$= \frac{1}{2\sqrt{2}}e^{-j45°}$

$\left|H\left(j\frac{6}{7}\right)\right|$ $\angle H\left(j\frac{6}{7}\right) = -45$

$H(j\omega) = \frac{\frac{3}{7}}{\frac{6}{7} + j\omega}$

$|H(j\omega)| = \frac{\frac{3}{7}}{\sqrt{\omega^2 + \left(\frac{6}{7}\right)^2}}$

$t_s = \frac{5}{|\text{Re pole}|} = \frac{5}{\left|-\frac{6}{7}\right|} = \frac{35}{6}$ sec

$\angle H(j\omega) = \angle\text{top} - \angle\text{bottom} = 0 - \text{Tan}^{-1}\left(\frac{\omega}{6/7}\right)$

Problem # 1

Recall: $M \longleftrightarrow \dfrac{M}{s}$

$$|M|\cos(\omega_0 t + \angle M) \longleftrightarrow |M|\left[\frac{\cos\angle M \, s - \omega_0 \sin\angle M}{s^2+\omega_0^2}\right]$$

$$u(t) = 10 - 7\cos\left(\tfrac{6}{7}t + 45^\circ\right) \longleftrightarrow U(s) = \frac{10}{s} - 7\left[\frac{\cos 45 \, s - \frac{6}{7}\sin 45^\circ}{s^2+\left(\frac{6}{7}\right)^2}\right]$$

$\frac{1}{\sqrt{2}}$

$$= \frac{\text{some numerator}}{s\left(s^2+\left(\frac{6}{7}\right)^2\right)} \leftarrow \left[\text{you do algebra at home!}\right]$$

$$Y = HU = \left[\frac{\frac{3}{7}}{s+\frac{6}{7}}\right]\frac{(\text{some numerator})}{s\left(s^2+\left(\frac{6}{7}\right)^2\right)} = \frac{A}{s} + \frac{B}{s+\frac{6}{7}} + \left[\frac{C}{s-j\frac{6}{7}} + *\right]$$

$$\Rightarrow y = A + Be^{-\frac{6}{7}t} + 2|C|\cos\left(\tfrac{6}{7}t + \angle C\right)$$

yss - computed earlier!

$$\Rightarrow A = \lim_{s\to 0} sY = \cancel{s}\, H(s)\frac{10}{\cancel{s}}\Big|_{s=0}$$

$$\doteq 10 \times H(0) \quad \left[\text{agrees with MOTF!!!}\right]$$

$$= 10\left(\tfrac{1}{2}\right)$$

Enough for an exam

$$C = \lim_{s\to j\frac{6}{7}} \left(s-j\tfrac{6}{7}\right)Y(s)$$

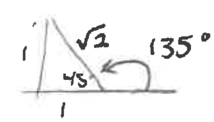
B can be found via: $B = \lim_{s\to -\frac{6}{7}} \left(s+\tfrac{6}{7}\right)Y(s)$

Lets find C:

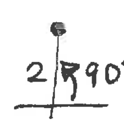
$$C = \lim_{s\to j\frac{6}{7}} \left(s-j\tfrac{6}{7}\right)Y(s) = \left(s-j\tfrac{6}{7}\right)H(s)\left[\frac{10}{7} - \frac{7}{\sqrt{2}}\left[\frac{s-\frac{6}{7}}{\underbrace{s^2+\left(\frac{6}{7}\right)^2}_{(s-j\frac{6}{7})(s+j\frac{6}{7})}}\right]\right]\Bigg|_{s=j\frac{6}{7}}$$

$$= H(s)\left[-\left(\frac{7}{\sqrt{2}}\right)\left(\frac{s-\frac{6}{7}}{s+j\frac{6}{7}}\right)\right]\Bigg|_{s=j\frac{6}{7}}$$

$$= - \frac{7}{\sqrt{2}} H\left(j\tfrac{6}{7}\right)\frac{j\frac{6}{7}-\frac{6}{7}}{j2\left(\frac{6}{7}\right)} \qquad \cancel{\sqrt{2}e^{j135^\circ}}$$



$$= 1e^{j180^\circ}\,\frac{7}{\cancel{\sqrt{2}}}\,H\left(j\tfrac{6}{7}\right)\frac{-1+j1}{j2} \qquad j2 - 2e^{j90^\circ}$$



Problem #1

$$C = 1\, e^{j180^\circ}\, \frac{7}{\cancel{\sqrt{2}}}\, |H(j\tfrac{6}{7})|\, e^{j\angle H(j\frac{6}{7})}\, \frac{\cancel{\sqrt{2}}\, e^{j135^\circ}}{2\, e^{j90^\circ}}$$

$$= \frac{7}{2}\, |H(j\tfrac{6}{7})|\, e^{j[\angle H(j\frac{6}{7}) + \underbrace{135^\circ - 90^\circ}_{45^\circ} + 180^\circ]}$$

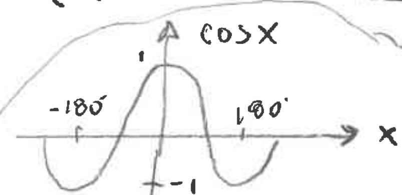
From the above A & C, we get the following for y_{ss}:

$$y_{ss} = A + 2|C| \cos\left(\tfrac{6}{7}t + \angle C\right)$$

$$= 10\,H(0) + 2\left(\tfrac{7}{2}\right) |H(j\tfrac{6}{7})| \cos\left(\tfrac{6}{7}t + 45^\circ + \angle H(j\tfrac{6}{7}) + 180^\circ\right)$$

$$= 10\,H(0) - 7\,|H(j\tfrac{6}{7})| \cos\left(\tfrac{6}{7}t + 45^\circ + \angle H(j\tfrac{6}{7})\right)$$

used $\cos(x \pm 180^\circ) = -\cos x$



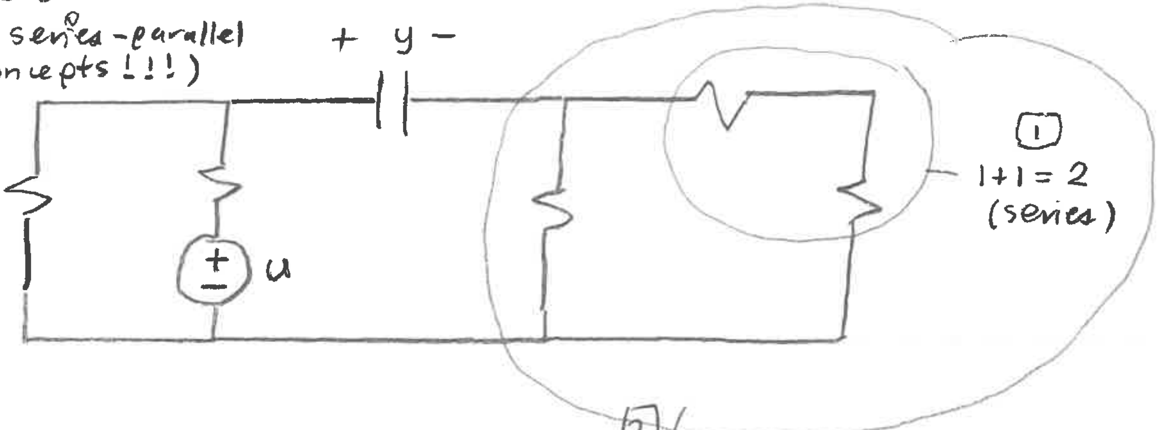
If we shift $\cos x$ left or right 180°, we get $-\cos x$!

This y_{ss} is precisely what was written above – 1st line; exploiting MOTF!

Problem # 1

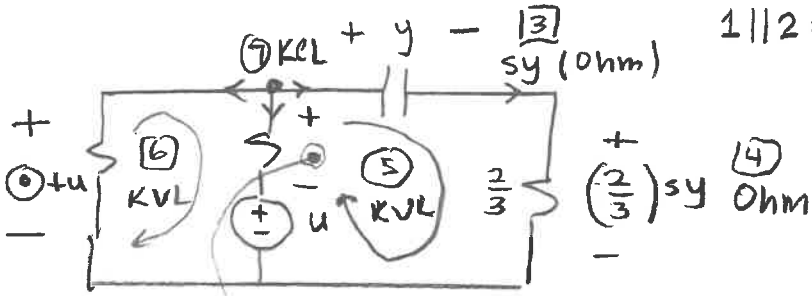
Lets derive H without source transformations:

(I will use series-parallel concepts!!!)



2 (parallel)

$$1||2 = \frac{(1)(2)}{1+2} = \frac{2}{3}$$



$$\odot = y + \left(\tfrac{2}{3}\right)sy - u \quad \text{(by KVL)}$$

7 KCL

$$\left[\frac{[1+\frac{2}{3}s]y-u+u}{1}\right] + \left[\frac{[1+\frac{2}{3}s]y-u}{1}\right] + sy = 0$$

$$\Rightarrow \quad y\Big[\underbrace{1+\tfrac{2}{3}s + 1 + \tfrac{2}{3}s + s}_{\frac{7}{3}s + \underbrace{2}_{\frac{6}{3}}}\Big] = u$$

$$\Rightarrow \quad y\,\frac{7}{3}\left[s + \frac{6}{7}\right] = u$$

$$\Rightarrow \quad \boxed{y = \underbrace{\left[\frac{\frac{3}{7}}{s+\frac{6}{7}}\right]}_{H} u}$$

H which is what we obtained earlier

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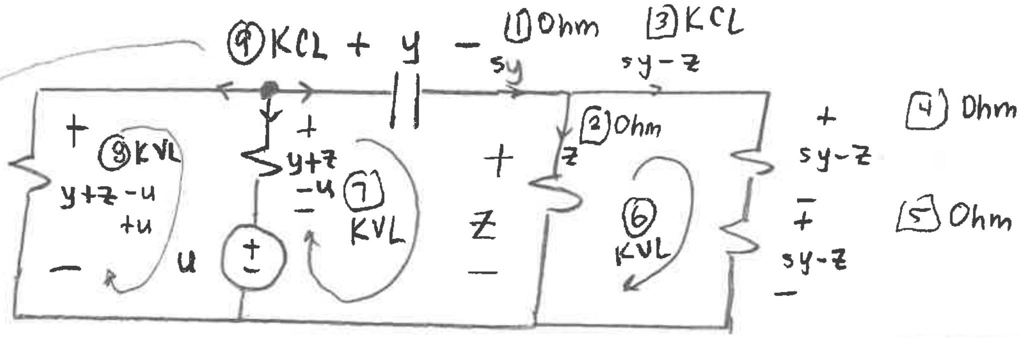
... what if we don't use source transformations or series-parallel concepts --- then what do we do? ...

Problem # 1

If we don't use source transformations or series-parallel concepts, then we are forced to introduce a variable z as follows:



⑥ KVL: $\boxed{z = (sy-z)+(sy-z)}$

$\Rightarrow 3z = 2sy$

$\Rightarrow z = \frac{2}{3}sy$

⑨ KCL:

$$\boxed{\left(\frac{y+z}{1}\right) + \left(\frac{y+z-u}{1}\right) + \left(sy\right) = 0}$$

using our 2 eqs in 2 unknowns (y, z)

$$y + \frac{2}{3}sy + y + \frac{2}{3}sy - u + sy = 0$$

$$\Rightarrow y\,[\underbrace{1 + \tfrac{2}{3}s + 1 + \tfrac{2}{3}s + s}_{\frac{7}{3}s+2}] = u$$

$$\Rightarrow y\,\frac{7}{3}\left[s + \frac{6}{7}\right] = u$$

$$\Rightarrow \boxed{y = \left[\frac{3/7}{s+\frac{6}{7}}\right]u}$$

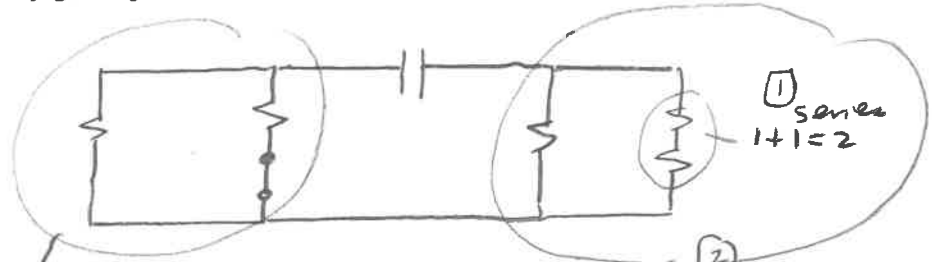
H − agrees with what we got earlier

... lets do a characteristic eq. & gain check ---

Problem # 1

It is always good to perform "CHECKS!"

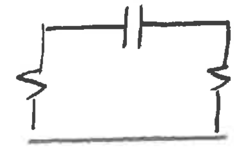
Φ Check: Set $u = 0$



1 series $1+1=2$

2 parallel $1 \| 2 = \frac{(1)(2)}{1+2} = \frac{2}{3}$

3 parallel $1 \| 1 = \frac{1}{2}$



$$\frac{1}{s}$$

$$\frac{1}{2} \qquad \frac{2}{3}$$

$$Z = \frac{1}{2} + \frac{2}{3} + \frac{1}{s}$$

$$= \frac{3+4}{6} + \frac{1}{s}$$

$$= \frac{7}{6} + \frac{1}{s}$$

$$= \frac{\frac{7}{6}s + 1}{s}$$

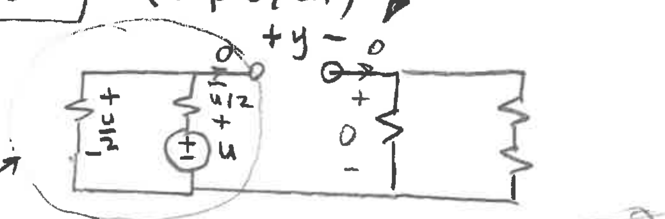
$$= \frac{\frac{7}{6}\left[s + \frac{6}{7}\right]}{s}$$

$$\Rightarrow \Phi = s + \frac{6}{7}$$

which agrees with what we got earlier!

Gain Checks: Be Careful: $y \neq 0$ for $s=0$

$\boxed{s=0}$ (cap open)



voltage divider

$\Rightarrow y = -\frac{u}{2} + u - 0 = \frac{1}{2}u \rightarrow H(0) = \frac{1}{2}$

$\boxed{s=\infty}$ (Cap shorted)

$\Rightarrow y = 0$

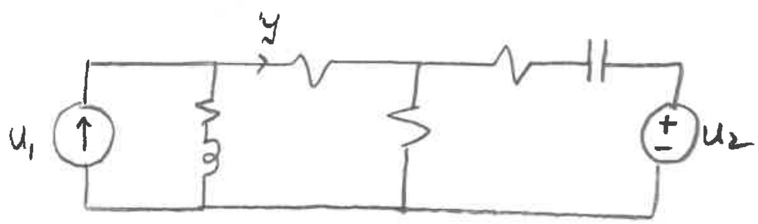
$\Rightarrow H(\infty) = 0$

These gains agree with $H = \frac{3/7}{s + 6/7}$

Problem #2

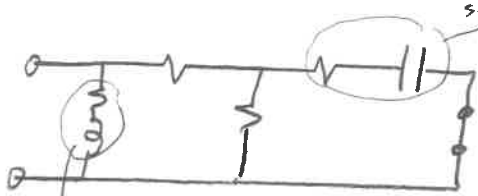
Determine poles, t_s, $H_1(0)$, $H_2(0)$, $H_1(\infty)$, $H_2(\infty)$.

(all $R = L = C = 1$)



Φ, poles, t_s: set $u_1 = 0$ (open) & $u_2 = 0$ (shorted)

① series $1 + \frac{1}{s} = \frac{s+1}{s}$



$s+1$ (series)

$\Rightarrow$

$$1 \,\|\, \frac{s+1}{s} = \frac{(1)\left(\frac{s+1}{s}\right)}{1 + \frac{s+1}{s}} = \frac{s+1}{s+s+1} = \frac{s+1}{2s+1}$$

$$\Rightarrow Z = (s+1) + (1) + \left(\frac{s+1}{2s+1}\right)$$

$$= s+2 + \left(\frac{s+1}{2s+1}\right)$$

$$= \frac{(s+2)(2s+1) + s+1}{2s+1}$$

$$= \frac{2s^2+s+4s+2+s+1}{2s+1}$$

$$= \frac{2s^2+6s+3}{2s+1}$$

$$= 2\left[\frac{s^2+3s+\frac{3}{2}}{2s+1}\right]$$

$$\Rightarrow \Phi = s^2+3s+\frac{3}{2} = 0$$ Characteristic polynomial

$$\Rightarrow \text{poles} = \frac{-3 \pm \sqrt{9-6}}{2} = \frac{-3 \pm \sqrt{3}}{2}$$

$$= \frac{-3+\sqrt{3}}{2}, \quad \frac{-3-\sqrt{3}}{2}$$

$$= -\sigma_1, \quad -\sigma_2$$

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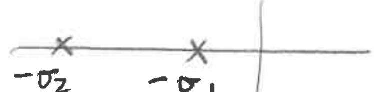


fast pole

slow pole $\Rightarrow$

$$t_s = \frac{5}{\sigma_1} = \frac{5}{\frac{3-\sqrt{3}}{2}}$$

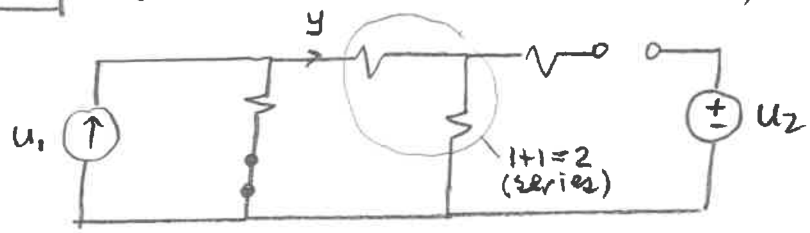
Problem # 2

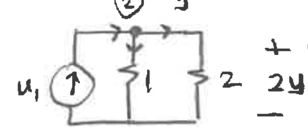
$$\Rightarrow \quad t_s = \frac{10}{3-\sqrt{3}}$$

Gains

$Z_L = sL = 0 \qquad Z_C = \frac{1}{sC} = \infty$

$\boxed{s = 0}$ (Ind shorted, Cap open)





② KCL $\Rightarrow \quad u_1 = \left(\frac{2y}{1}\right) + (y)$

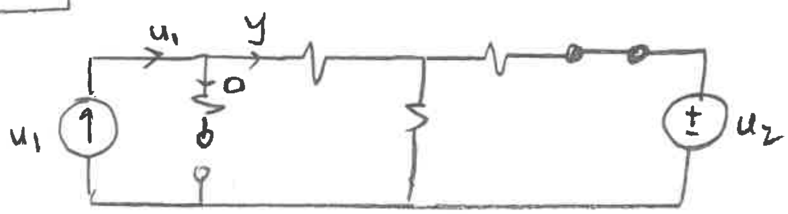
$= 3y$

$\Rightarrow \quad y = \frac{1}{3} u_1 + 0\, u_2$

$$\boxed{H_1(0) = \tfrac{1}{3} \quad H_2(0) = 0}$$

$Z_L = sL = \infty \qquad Z_C = \frac{1}{sC} = 0$

$\boxed{s = \infty}$ (Ind open, Cap shorted)



$\Rightarrow \quad y = u_1$

$= (1)u_1 + (0)u_2$

$$\boxed{H_1(\infty) = 1 \quad H_2(\infty) = 0}$$

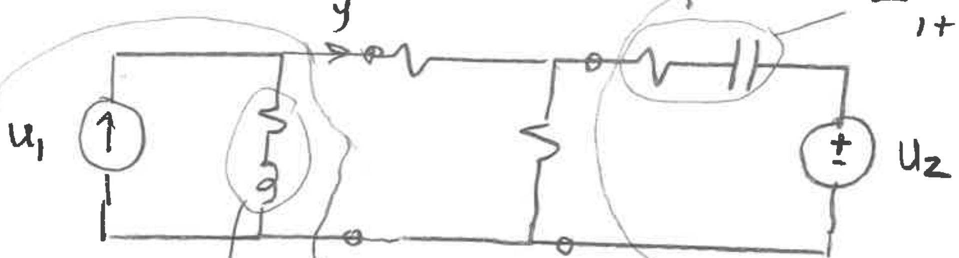
Now lets do all of the above using our

bread & butter KVL, KCL, Ohm....

or clever circuit analysis ---

Problem #2

1st some clever circuit analysis:

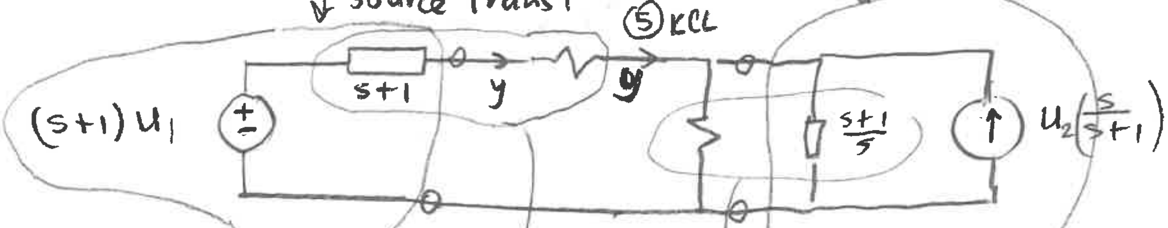


[3] series $1 + \frac{1}{s} = \frac{s+1}{s}$

[1] $s+1$ (series)

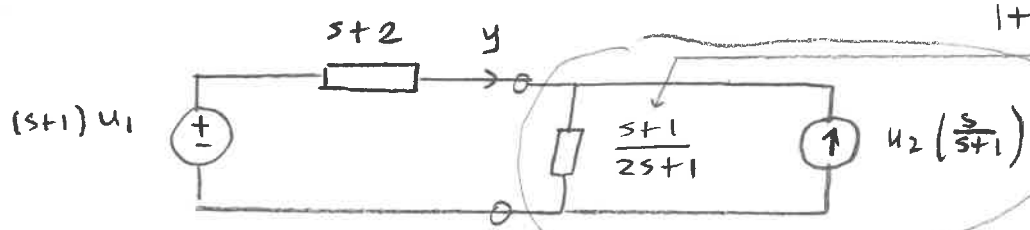
[4] source transf

[2] source transf

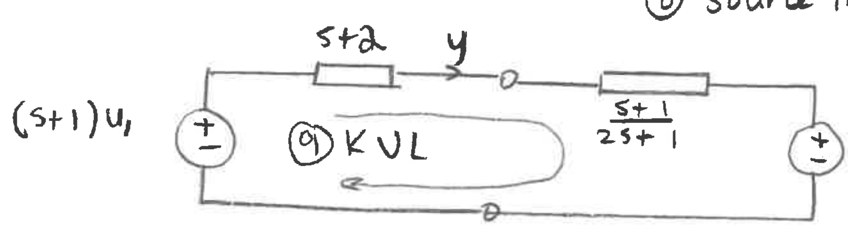


(6) series
$(s+1)+(1) = s+2$

(7) parallel
$1 \| \frac{s+1}{s} = \frac{(1)\left(\frac{s+1}{s}\right)}{1 + \frac{s+1}{s}} = \frac{s+1}{2s+1}$



(8) source transf



$u_2\left(\frac{s}{2s+1}\right) = u_2\left(\frac{s}{\cancel{s+1}}\right)\left(\frac{\cancel{s+1}}{2s+1}\right)$

(9) KVL: $\boxed{(s+1)u_1 = (s+2)y + \left(\frac{s+1}{2s+1}\right)y + u_2\left(\frac{s}{2s+1}\right)}$

$\overset{\text{algebra}}{\Longrightarrow} y\left[s+2 + \frac{s+1}{2s+1}\right] = (s+1)u_1 + \left(\frac{-s}{2s+1}\right)u_2$

$\Longrightarrow y\left[\underbrace{(s+2)(2s+1)}_{2s^2+s+4s+2} + s+1\right] = (s+1)(2s+1)u_1 + (-s)u_2$

90|280

Problem # 2

$$\Rightarrow y\left[2s^2 + s + 4s + 2 + s + 1\right] = (s+1)(2s+1)u_1 + (-s)u_2$$

$$\Rightarrow y\left[2s^2 + 6s + 3\right] = (s+1)(2s+1)u_1 + (-s)u_2$$

$$= 2\left(s^2 + 3s + \tfrac{3}{2}\right)$$

$$\Rightarrow \boxed{y = \underbrace{\left[\frac{\frac{1}{2}(s+1)(2s+1)}{s^2+3s+\frac{3}{2}}\right]}_{H_1} u_1 + \underbrace{\left[\frac{-s}{s^2+3s+\frac{3}{2}}\right]}_{H_2} u_2}$$

Note: $\Phi = s^2 + 3s + \frac{3}{2}$ as obtained earlier!

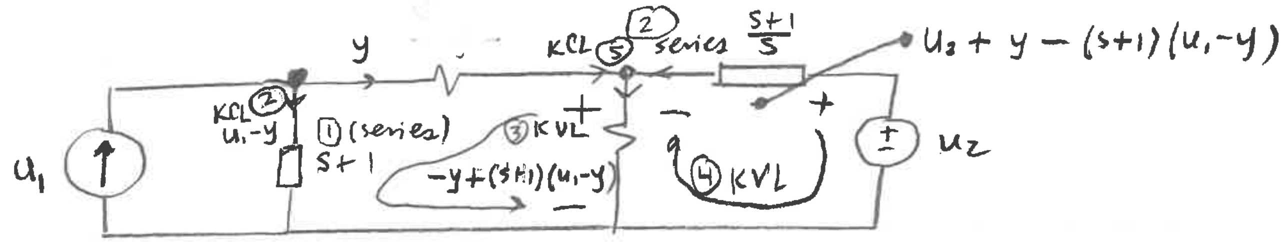
$H_1(0) = \dfrac{(\frac{1}{2})(1)(1)}{\frac{3}{2}} = \frac{1}{3}$ $\qquad H_2(0) = 0$ " " " !

$H_1(\infty) = (\frac{1}{2})(2) = 1$ $\qquad H_2(\infty) = 0$ " " " !

Problem # 2

Now lets not be clever.

Lets just use KVL, KCL, Ohm (+ series concepts) =



⑤ KCL

$$\boxed{(y) + \left[u_2 + y - (s+1)(u_1 - y)\right]\frac{s}{s+1} = \left(\frac{-y+(s+1)(u_1-y)}{1}\right)}$$

$$\Rightarrow y\left[1 + \frac{s}{s+1} + s + 1 + s + 1\right] = u_1\left[s+1+s\right] - u_2\left[\frac{s}{s+1}\right]$$

$2s + 3$

$$\Rightarrow y\left[\frac{s}{s+1} + 2s + 3\right] = u_1\left[2s+1\right] - u_2\left[\frac{s}{s+1}\right]$$

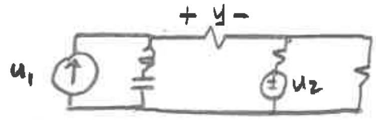
$$\Rightarrow y\left[s + \underbrace{(2s+3)(s+1)}_{2s^2+2s+3s+3}\right] = u_1(s+1)(2s+1) - s u_2$$

$$\Longrightarrow y\left[\underbrace{2s^2 + 5s + 3}_{2(s^2+3s+\frac{3}{2})}\right] = (s+1)(2s+1)u_1 + (-s)u_2$$

$$\Longrightarrow \boxed{y = \left[\frac{(s+1)(s+\frac{1}{2})}{s^2+3s+\frac{3}{2}}\right]u_1 + \left[\frac{-\frac{1}{2}s}{s^2+3s+\frac{3}{2}}\right]u_2}$$

which agrees with what we obtained earlier!

Problem #3



Determine $H_{1,2}$, ts, diff eq, yss

(all R = L = C = 1)

Since our circuit consists of R's, L's, C's & indep sources, we know the circuit must be stable (i.e. all poles in left half s-plane = Re s < 0)

Recall:

$u_1 = 3 - 4\cos(t - 50°)$

$u_2 = -8 - 2\sin(t + 30°) + 200\cos(100t + 95°)$

Given this, it follows that

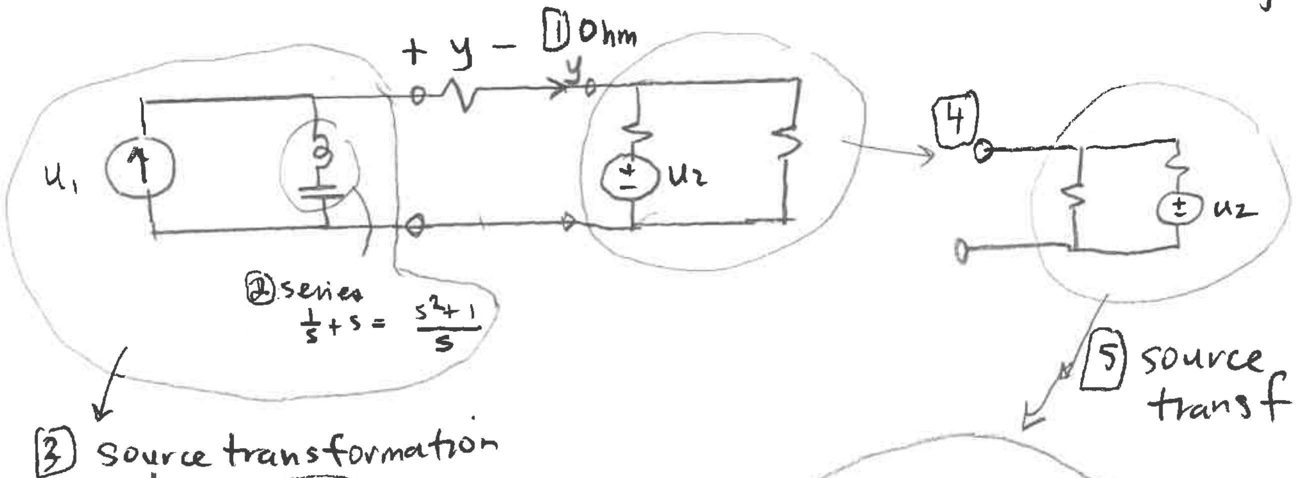
$$y_{ss} = y_{1ss} + y_{2ss}$$

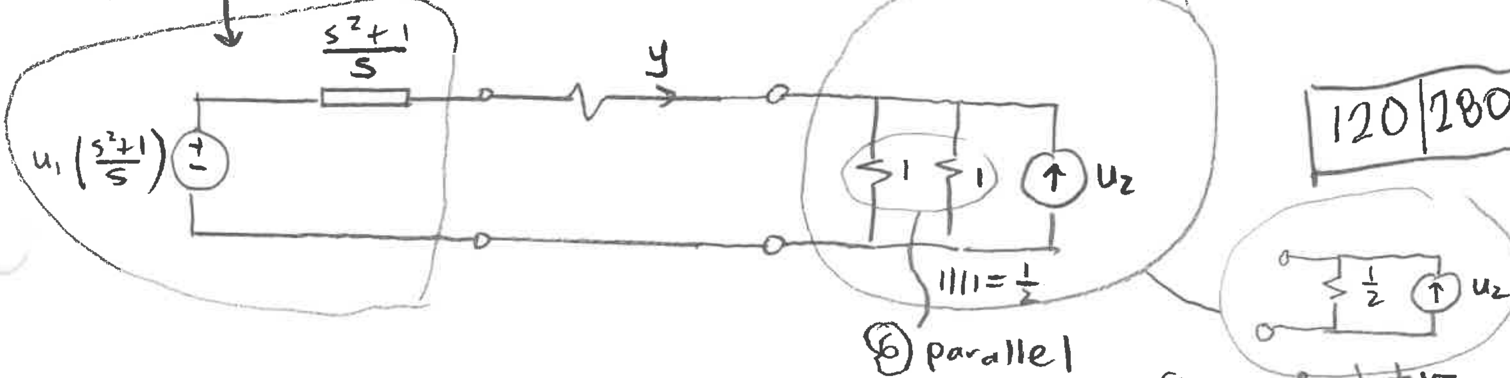
MOTF

$$y_{1ss} = 3H_1(0) - 4|H_1(j1)|\cos(t - 50° + \angle H_1(j1))$$

$$y_{2ss} = -8H_2(0) - 2|H_2(j1)|\sin(t + 30° + \angle H_2(j1)) + 200|H_2(j100)|\cos(100t + 95° + \angle H_2(j100))$$

Now lets find $H_{1,2}$ ... 1st by some "slightly clever" circuit analysis ...





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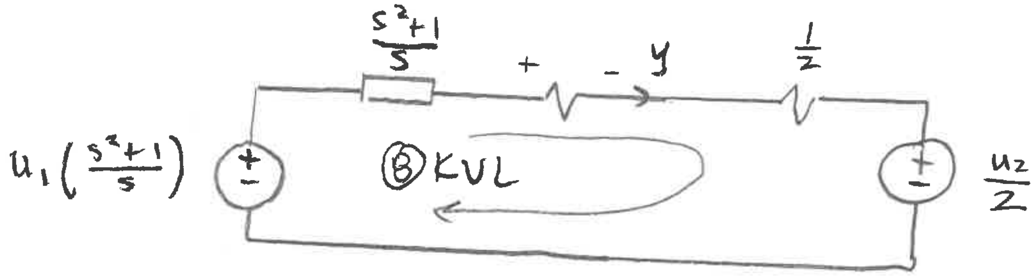
Problem # 3



$$\text{B KVL} = \quad u_1\left(\frac{s^2+1}{s}\right) = \left(\frac{s^2+1}{s}\right)y + y + \frac{1}{2}y + \frac{u_2}{2}$$

$$\Rightarrow y\left[\frac{s^2+1}{s} + \overbrace{\left(1+\frac{1}{2}\right)}^{3/2}\right] = \left[\frac{s^2+1}{s}\right]u_1 + \left[-\frac{1}{2}\right]u_2$$

$$\Rightarrow y\left[s^2+1+\frac{3}{2}s\right] = \left[s^2+1\right]u_1 + \left[-\frac{1}{2}s\right]u_2$$

$$\Rightarrow \boxed{y = \underbrace{\left[\frac{s^2+1}{s^2+\frac{3}{2}s+1}\right]}_{H_1}u_1 + \underbrace{\left[\frac{-\frac{1}{2}s}{s^2+\frac{3}{2}s+1}\right]}_{H_2}u_2}$$

$$\Rightarrow \ddot{y} + \frac{3}{2}\dot{y} + y = \ddot{u}_1 + u_1 - \frac{1}{2}\dot{u}_2$$

$$\Rightarrow \Phi = s^2 + \frac{3}{2}s + 1 = 0 \quad \text{(characteristic polynomial)}$$

$$\Rightarrow \text{poles} = \frac{-\frac{3}{2} \pm \sqrt{\overbrace{\frac{9}{4} - \underbrace{4(1)}_{\frac{16}{4}}}^{-\frac{7}{4}}}}{2} = -\frac{3}{4} \pm j\frac{\sqrt{7}}{4}$$

$$\Rightarrow t_s = \frac{5}{|\text{Re poles}|}$$

$$= \frac{5}{\frac{3}{4}} = \frac{20}{3}\ \text{sec}$$

Gains Needed in y_{ss}:

$H_1(0) = \underline{1}$ $\qquad H_1(j1) = 0 \Leftarrow$ since H_1 has zero @ $\pm j1$

$H_2(0) = 0 \Leftarrow$ since H_2 has a zero @ $s=0$ $\qquad \frac{\phi j1}{\phi -j1}$

$$H_2(j1) = \frac{-\frac{1}{2}j1}{-1+j\frac{3}{2}+1} = \frac{-j\frac{1}{2}}{j\frac{3}{2}} = -1 = \underbrace{1}_{|H_2(j1)|}e^{j\overbrace{180^\circ}^{\angle H_2(j1)}}$$

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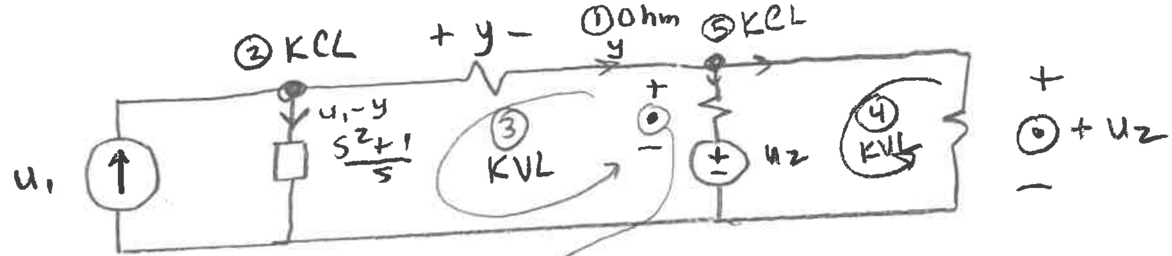
$$H_2(j100) \cong \frac{-\frac{1}{2}s}{s^2} = \left.\frac{-\frac{1}{2}}{s}\right|_{s=j100} = \frac{\frac{1}{2}e^{j180}}{100e^{j90}} = \underbrace{\frac{1}{200}}_{|H_2(j100)|}e^{j\overbrace{90^\circ}^{\angle H_2(j100)}}$$

Problem # 3

Now lets just use straight KVL, KCL, Ohm:

(with ind & cap in series: $Z_{LC} = \frac{1}{s} + s = \frac{s^2+1}{s}$)



$$\odot = -y + \left(\frac{s^2+1}{s}\right)(u_1 - y) - u_2$$

⑤ KCL:

$$\boxed{(y) = \left(\frac{-y + \left(\frac{s^2+1}{s}\right)(u_1 - y) - u_2}{1}\right) + \left(\frac{-y + \left(\frac{s^2+1}{s}\right)(u_1 - y) - u_2 + u_2}{1}\right)}$$

$$\Rightarrow y\left[1 + 1 + \frac{s^2+1}{s} + 1 + \frac{s^2+1}{s}\right] = \left[\frac{s^2+1}{s} + \frac{s^2+1}{s}\right]u_1 - u_2$$

$$\Rightarrow y\left[2\left(\frac{s^2+1}{s}\right) + 3\right] = \left[2\left(\frac{s^2+1}{s}\right)\right]u_1 - u_2$$

$$\Rightarrow y\left[\underbrace{2s^2 + 2 + 3s}_{2(s^2 + \frac{3}{2}s + 1)}\right] = \left[2(s^2+1)\right]u_1 - s\,u_2$$

$$\Rightarrow \boxed{y = \underbrace{\left[\frac{s^2+1}{s^2 + \frac{3}{2}s + 1}\right]}_{H_1}u_1 + \underbrace{\left[\frac{-\frac{1}{2}s}{s^2 + \frac{3}{2}s + 1}\right]}_{H_2}u_2}$$

– this agrees with what we obtained earlier!

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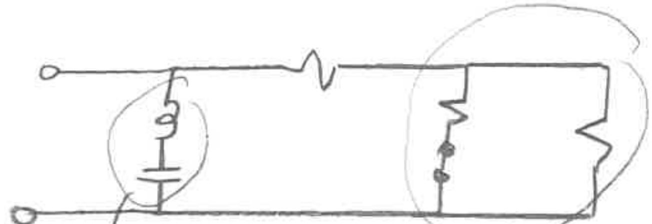
Problem # 3

It's Always Good to perform "CHECKS!"

Lets now do a characteristic polynomial & gain check.

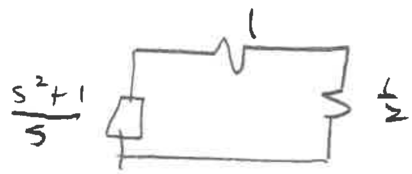
Φ check = set $u_1 = 0$ (open)
$u_2 = 0$ (short)



parallel

(2) series
$s + \frac{1}{s} = \frac{s^2+1}{s}$

(1) $1 \| 1 = \frac{1}{2}$



$$\Rightarrow Z = \frac{s^2+1}{s} + 1 + \frac{1}{2}$$
$$= \frac{s^2+1}{s} + 3/2$$
$$= \frac{s^2+1+3/2\, s}{s}$$

$$\Rightarrow \Phi = s^2 + \tfrac{3}{2}s + 1$$

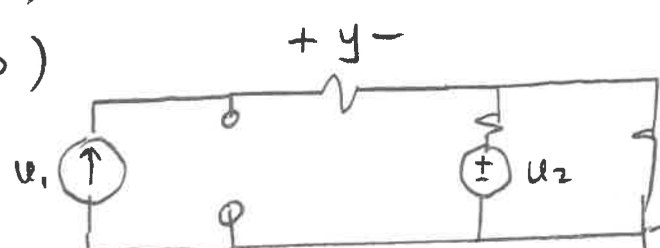
just as we got earlier! ✓

Gain Checks:

$\boxed{s = 0}$ (short ind, open cap)

$\boxed{s = \infty}$ (open ind, short cap)

either way, we have



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$$\Rightarrow y = u_1 = 1\,u_1 + 0\,u_2$$

$$\Rightarrow \boxed{\begin{aligned} H_1(0) &= H_1(\infty) = 1 \\ H_2(0) &= H_2(\infty) = 0 \end{aligned}}$$

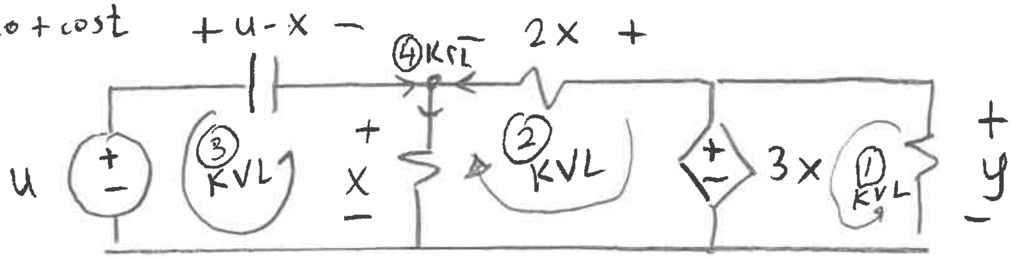
$\Rightarrow$ these agree with $H_{1,2}$ obtained earlier!

Problem # 4

Determine H, diff eq, y, y_{ss}

(all $R=L=C=1$)

$u = 10 + \cos t$



1st note that $y = 3x$ (by KVL)

or $\boxed{x = \frac{1}{3} y}$

2 eqs in 2 unknowns (y, x)

④KCL =

$$\boxed{[s(u-x)] + \left[\frac{2x}{1}\right] = \left[\frac{x}{1}\right]}$$

-- but it is easy to get rid of x since $x = \frac{1}{3} y$

$$\Rightarrow \quad su = x\,[s - 2 + 1]$$

$$= x\,[s-1]$$

$$= \frac{1}{3} y\,[s-1]$$

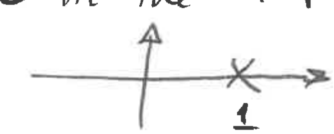
$$\Rightarrow \quad \boxed{y = \underbrace{\left[\frac{3s}{s-1}\right]}_{H} u}$$

Note that $H(0) = 0$! since H has a zero at $s = 0$

Why? At dc, cap is open & u has no impact on y

$$\Rightarrow \quad \dot{y} - y = 3\dot{u}$$

Note: system H is unstable with a pole in the RHP (right half s-plane) at $s = 1$



$u = 10 + \cos t$

$\updownarrow$

$$U = \frac{10}{s} + \frac{s}{s^2+1} = \frac{10(s^2+1) + s^2}{s(s^2+1)} = \frac{11s^2 + 10}{s(s^2+1)}$$

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Problem #4

$$Y = HU = \left[\frac{3\cancel{s}}{s-1}\right]\left[\frac{11s^2+10}{\cancel{s}(s^2+1)}\right]$$

$$= \left[\frac{3}{s-1}\right]\left[\frac{11s^2+10}{\underbrace{s^2+1}_{(s-j1)(s+j1)}}\right]$$

$$= \frac{A}{s-1} + \left[\frac{B}{s-j1} + *\right]$$

$$y = Ae^{t} + 2|B|\cos(t + \angle B) = y_{ss}$$

since nothing in y goes away as $t \to \infty$!

unstable growing exponential (dominant in steady state!)

$y_{ss} \cong Ae^{t}$ after a sufficiently long time

from MOTF this must equal

$|H(j1)| \cos(t + \angle H(j1))$

due to sinusoidal component of u

$u_{sinusoid} = \cos t$

$$A = \lim_{s\to 1} (s-1)Y$$

$$= \cancel{(s-1)}\left[\frac{3s}{\cancel{s-1}}\right]\left[\frac{11s^2+10}{s(s^2+1)}\right]\Bigg|_{s=1}$$

$$= \frac{3(1)[11+10]}{(1)(1+1)}$$

$$= \frac{3(21)}{2}$$

$$\Longrightarrow B = \frac{|H(j1)|}{2} e^{j\angle H(j1)}$$

$3\angle 90^\circ$

$$H(j1) = \frac{3j1}{j1-1} = \frac{j3}{-1+j1}$$

$1 \quad 45 \quad 135^\circ$

$$= \frac{3e^{j90^\circ}}{\sqrt{2}e^{j135^\circ}} = \underbrace{\frac{3}{\sqrt{2}}}_{|H(j1)|} e^{j\overbrace{(90-135)}^{\angle H(j1)}}$$

Note: since $u = 10 + \cos t$

From MOTF & Partial fraction Expansion (PFE) concepts we know

from MOTF

from PFE concepts

$$y_{ss} = \underbrace{10H(0)}_{\text{zero since } H(0)=0} + |H(j1)|\cos(t+\angle H(j1)) + Ae^{t}$$

zero since $H(0) = 0$ [since H has a zero at $s=0$]

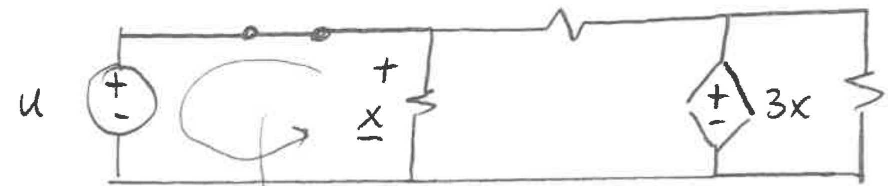
$\boxed{170|280}$

Problem #4

Lets do a gain check for $s = \infty$:

$Z_C = \frac{1}{sC} = 0$

$\boxed{s = \infty}$ (Cap is shorted)



$y = 3x = 3u$

① KVL: $\boxed{x = u}$

$H(\infty) = 3$ ✓

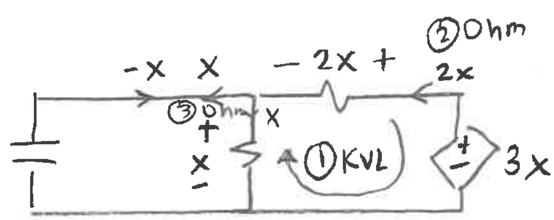
- this agrees with

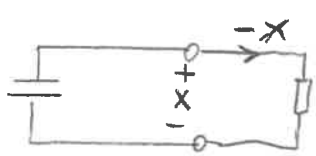
$$H = \frac{3s}{s-1}$$

obtained earlier!

Lets do a characteristic eq check:

Set $u = 0$





$\leftarrow R_{th} = -1$

$$Z = \frac{1}{s} - 1 = \frac{1-s}{s} = -\left[\frac{s-1}{s}\right]$$

$\Rightarrow \Phi = s - 1$

as obtained earlier!

Note:

It is always good to perform "Checks"!

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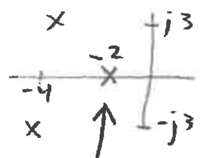
Problem # 5

Determine t_s, y_{ss}, y

$u = -5 - 6\cos(0.01t - 45^\circ) + 7\sin(100t + 60^\circ)$

$$H(s) = \frac{(s+0.01)(s^2+s+1)}{(s+2)(s^2+8s+25)(s^2-2s+4)}$$

poles: -2 stable; $-4 \pm j3$ stable; $1 \pm j\sqrt{3}$ (unstable)



slow pole @ $s=-2$ $\Rightarrow t_s = \frac{5}{|-2|} = \frac{5}{2} = 2.5$ sec

From MOTF

$$y_{ss} = -5H(0) - 6|H(j0.01)|\cos(0.01t - 45^\circ + \angle H(j0.01)) + 7|H(j100)|\sin(100t + 60^\circ + \angle H(j100))$$

$+$ growing exponential sinusoidal instability $2|D|e^{t}\cos(\sqrt{3}t + \angle D)$

from partial fraction expansion ideas

dominant in steady state

$$H(0) = \frac{(0.01)(1)}{(2)(25)(4)}$$

$$H(j0.01) \cong \frac{(j0.01 + 0.01)(1)}{(2)(25)(4)} = \frac{(1)(0.01)}{(2)(25)(4)}(1+j1)$$

$\sqrt{2}e^{j45}$

$$= \frac{(1)(0.01)\sqrt{2}}{(2)(25)(4)} e^{j45^\circ}$$

$|H(j0.01)|$

$\angle H(j0.01)$

$$H(j100) \cong \frac{s^3}{s^5} = \frac{1}{s^2}\Big|_{s=j100} = \frac{1}{(100e^{j90})^2} = 10^{-4} e^{-j180}$$

$|H(j100)| \cong 10^{-4}$

$\angle H(j100) \cong -2(90) = -180^\circ$

Now lets find y ---

Problem # 5

$u = -5 - 6\cos(0.01t - 45°) + 7\sin(100t + 60°)$

$\Updownarrow$

$$U = -\frac{5}{s} - \frac{\square s + \square}{s^2 + 10^{-4}} + \frac{\square s + \square}{s^2 + 10^4}$$

$$= \frac{\text{some numerator}}{s(s^2+10^{-4})(s^2+10^4)}$$ ← you do the algebra @ home!

$$Y = HU = \left[\frac{(s+0.01)(s^2+s+1)}{(s+2)(s^2+8s+25)(s^2-2s+4)}\right]\left[\frac{\text{some numerator}}{s(s^2+10^{-4})(s^2+10^4)}\right]$$

$-4+j3 \quad 1+j\sqrt{3} \quad j0.01 \quad j100$

$$= \frac{A}{s} + \frac{B}{s+2} + \left[\frac{C}{s+4-j3} + *\right] + \left[\frac{D}{s-1-j\sqrt{3}} + *\right]$$

$$+ \left[\frac{E}{s-j0.01} + *\right]$$

$$+ \left[\frac{F}{s-j100} + *\right]$$

$-5H(0)$

growing instability (dominates yss)

$$y = A + Be^{-2t} + 2|C|e^{-4t}\cos(3t + \angle C) + 2|D|e^{t}\cos(\sqrt{3}t + \angle D)$$

$$+ 2|E|\cos(0.01t + \angle E)$$

$$+ 2|F|\cos(100t + \angle F)$$

These 3 terms are in yss (along with instability)
The 3 terms were found earlier using MOTF!

$$\Longrightarrow \quad A = \lim_{s\to 0} sY = \cancel{s}\,H\left(\frac{-5}{\cancel{s}}\right)\Big|_{s=0} = -5H(0)$$

$$\boxed{D = \lim_{s\to 1+j\sqrt{3}} (s-1-j\sqrt{3})Y}$$

$$E = \lim_{s\to j0.01} (s-j0.01)Y$$

$$F = \lim_{s\to j100} (s-j100)Y$$

$-6|H(j0.01)|\cos(0.01t - 45° + \angle H(j0.01))$

$+ 7|H(j100)|\sin(100t + 60 - \angle H(j100))$

$= 6|H(j0.01)|\cos(0.01t - 45 + \angle H(j0.01) - 180)$

$+ 7|H(j100)|\cos(100t + 60 - \angle H(j100) - 90°)$

From these $E = \frac{6|H(j0.01)|}{2}e^{j(-45° + \angle H(j0.01) - 180°)}$

$F = \frac{7|H(j100)|}{2}e^{j(60 - \angle H(j100) - 90°)}$

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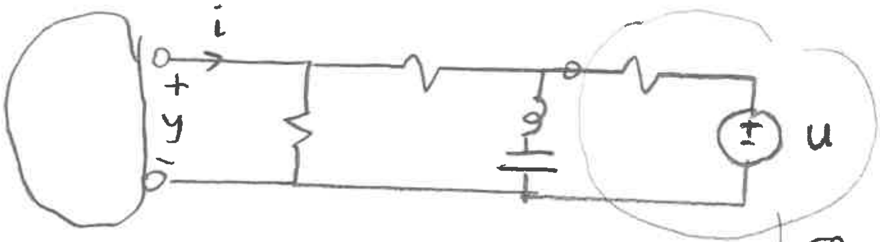
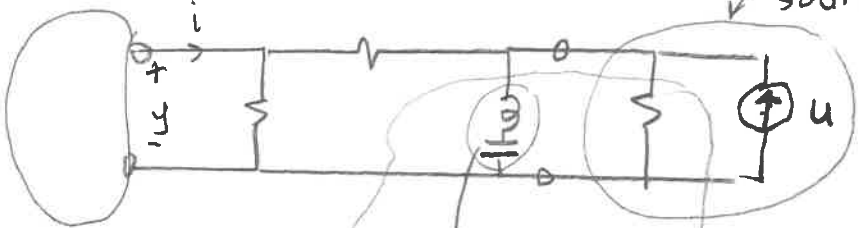
Problem # 6

Determine an s-domain Thevenin equivalent at y.
(specify: Z_{th} & V_{th} !) (all R = L = C = 1)

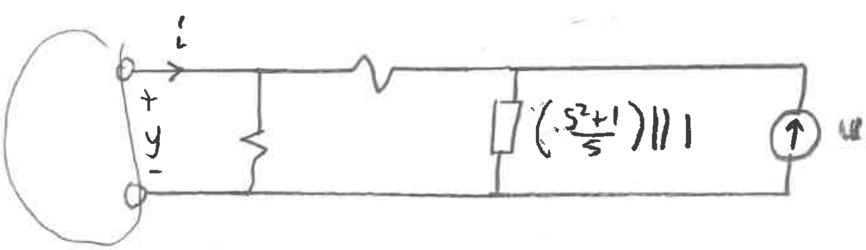


[1] source transformation



[2] series
$s + \frac{1}{s} = \frac{s^2+1}{s}$

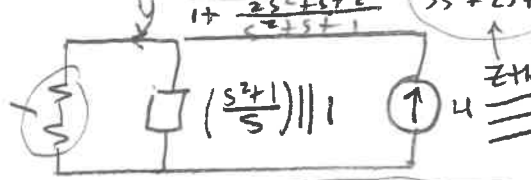
$$\left(\frac{s^2+1}{s}\right) \| 1 = \frac{\left(\frac{s^2+1}{s}\right)(1)}{\frac{s^2+1}{s} + 1} = \frac{s^2+1}{s^2+s+1}$$



$$1 + \left(\frac{s^2+1}{s}\right) \| 1 = 1 + \frac{s^2+1}{s^2+s+1} = \frac{s^2+s+1+s^2+1}{s^2+s+1} = \frac{2s^2+s+2}{s^2+s+1}$$

$$\Rightarrow \boxed{Z_{th} = 1 \,\|\, \left[1 + \left(\frac{s^2+1}{s}\right) \| 1\right]} = 1 \,\|\, \frac{2s^2+s+2}{s^2+s+1} = \frac{\frac{2s^2+s+2}{s^2+s+1}}{1 + \frac{2s^2+s+2}{s^2+s+1}} = \frac{2s^2+s+2}{3s^2+2s+3}$$

Z_{th}



$V_{th} = y \big|_{i=0}$

current division

$$\boxed{V_{th} = \left[\frac{\left(\frac{s^2+1}{s}\right) \| 1}{2 + \left(\frac{s^2+1}{s}\right) \| 1}\right] u}$$

$$= \left[\frac{\frac{s^2+1}{s^2+s+1}}{2 + \frac{s^2+1}{s^2+s+1}}\right] u = \left[\frac{s^2+1}{2s^2+2s+2+s^2+1}\right] u = \left[\frac{s^2+1}{3s^2+2s+3}\right] u$$

V_{th}

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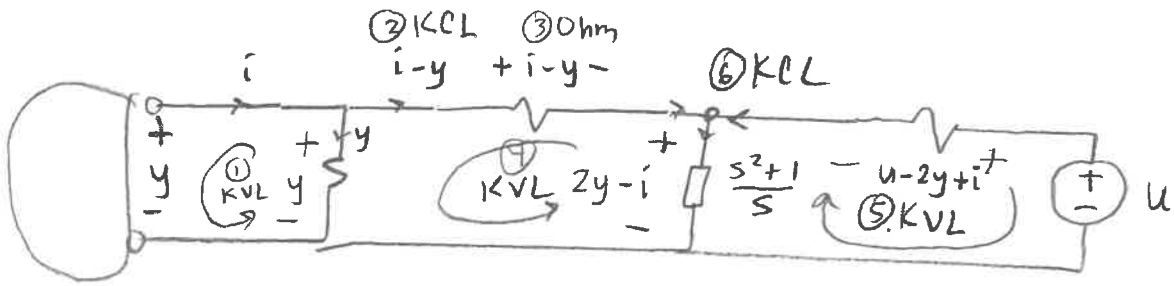
Problem # 6

Solution via KVL, KCL, Ohm:



⑥ KCL:

$$\boxed{(i-y) + \left(\frac{u-2y+i}{1}\right) = (2y-i)\left(\frac{s}{s^2+1}\right)}$$

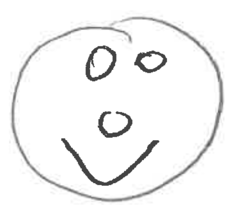
$$\Rightarrow \quad i\left[1+1+\frac{s}{s^2+1}\right] + u = y\left[1+2+\frac{2s}{s^2+1}\right]$$

$$\Rightarrow y\left[3+\frac{2s}{s^2+1}\right] = \left[2+\frac{s}{s^2+1}\right]i + u$$

$$\Rightarrow y\left[3s^2+3+2s\right] = \left[2s^2+2+s\right]i + (s^2+1)u$$

$$\Rightarrow \boxed{y = \underbrace{\left[\frac{2s^2+s+2}{3s^2+2s+3}\right]}_{Z_{th}} i + \underbrace{\left[\frac{s^2+1}{3s^2+2s+3}\right]}_{V_{th}} u}$$

Zth & Vth here agree with that obtained on previous page



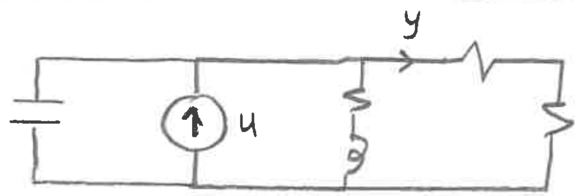
220|280

Compute all coefficients in yss approximately!

Problem # 7

Determine H, y_{ss}, t_s

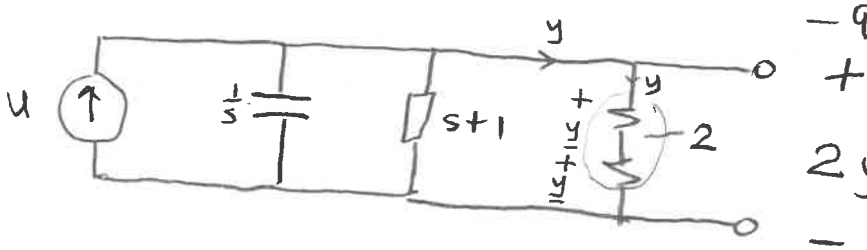
(all $R=L=C=1$)

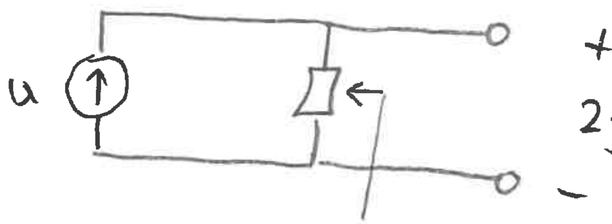


$$u = -7 - 8\cos\left(\sqrt{\tfrac{3}{2}}\,t - 30^\circ\right) - 9\sin(100t + 53^\circ)$$

solution: $\xRightarrow{\text{since cct is stable}}$

$$y_{ss} = -7H(0) - 8\left|H\left(j\sqrt{\tfrac{3}{2}}\right)\right|\cos\left(\sqrt{\tfrac{3}{2}}\,t - 30^\circ + \angle H\left(j\sqrt{\tfrac{3}{2}}\right)\right) - 9\left|H(j100)\right|\sin\left(100t + 53^\circ + \angle H(j100)\right)$$





$$Z = \left(\tfrac{1}{s}\right) \| (s+1) \| (2)$$

$$\xRightarrow{\text{Ohm}} \quad 2y = Zu = \left[\left(\tfrac{1}{s}\right) \| (s+1) \| (2)\right] u$$

$$\boxed{y = \underbrace{\tfrac{1}{2}\left[\left(\tfrac{1}{s}\right) \| (s+1) \| 2\right]}_{H} u}$$

$$\frac{1}{s} \| (s+1) = \frac{\frac{s+1}{s}}{\frac{1}{s} + s + 1} = \frac{s+1}{s^2+s+1}$$

$$\left[\frac{1}{s} \| (s+1)\right] \| 2 = \frac{2\left[\frac{s+1}{s^2+s+1}\right]}{2 + \frac{s+1}{s^2+s+1}} = \frac{2(s+1)}{2s^2+3s+3} = \frac{\cancel{2}(s+1)}{\cancel{2}\left[s^2+\frac{3}{2}s+\frac{3}{2}\right]}$$

$$\boxed{H = \frac{\frac{1}{2}(s+1)}{s^2+\frac{3}{2}s+\frac{3}{2}}}$$

Note: $H(0) = 1/3$, $H(\infty) = 0$

gains needed in yss:

$$H(0) = \frac{\left(\frac{1}{2}\right)(1)}{\left(\frac{3}{2}\right)} = 1/3$$

$$H\left(j\sqrt{\tfrac{3}{2}}\right) = \frac{\frac{1}{2}\left(j\sqrt{\frac{3}{2}}+1\right)}{-\frac{3}{2} + j\frac{3}{2}\sqrt{\frac{3}{2}} + \frac{3}{2}} = \frac{\frac{1}{2}\left(1+j\sqrt{\frac{3}{2}}\right)}{j\frac{3}{2}\sqrt{\frac{3}{2}}}$$

Problem # 7

$$H(j\sqrt{\tfrac{3}{2}}) = \frac{\frac{1}{2}\left(1+j\sqrt{\frac{3}{2}}\right)}{j\,\frac{3}{2}\sqrt{\frac{3}{2}}}$$

$$= \underbrace{\frac{\frac{1}{2}\sqrt{1+\frac{3}{2}}}{\frac{3}{2}\sqrt{\frac{3}{2}}}}_{|H(j\sqrt{\frac{3}{2}})|} e^{j\overbrace{\left(\tan^{-1}\left(\frac{\sqrt{3/2}}{1}\right)-90^\circ\right)}^{\angle H(j\sqrt{\frac{3}{2}})}}$$

$$H(j100) \cong \frac{\frac{1}{2}s}{s^2} = \left.\frac{\frac{1}{2}}{s}\right|_{s=j100} = \frac{1/2}{100e^{j90^\circ}} = \frac{1}{200}e^{-j90^\circ}$$

$$|H(j100)| \cong \frac{1}{200}$$

$$\angle H(j100) \cong -90^\circ$$

Now lets get Φ, poles, t_s =

$$\Phi = s^2 + \frac{3}{2}s + \frac{3}{2} = 0$$

$$\Rightarrow \text{poles} = \frac{-\frac{3}{2} \pm \sqrt{\frac{9}{4}-6}}{2} \qquad \frac{9}{4}-\frac{24}{4} = \frac{-15}{4}$$

$$= -\frac{3}{4} \pm j\frac{\sqrt{15}}{4}$$

$$\Rightarrow t_s = \frac{5}{|\text{Re poles}|} = \frac{5}{\frac{3}{4}} = \frac{20}{3}$$

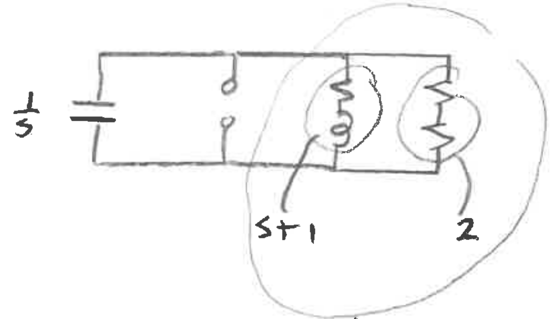
|240|280

Lets do a characteristic poly & gain check ...

Problem # 7

It's always good to perform CHECKS!

Φ Check: Set $u = 0$



$(s+1) \| 2 = \dfrac{2(s+1)}{s+3}$

$$\Rightarrow Z = \frac{1}{s} + \frac{2(s+1)}{s+3}$$

$$= \frac{s+3+2(s+1)s}{s(s+3)}$$

$$= \frac{s+3+2s^2+2s}{s(s+3)}$$

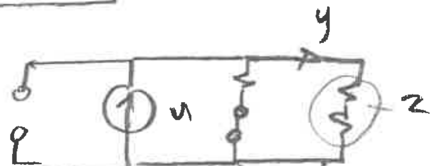
$$= \frac{2s^2+3s+3}{s(s+3)}$$

$$= 2\left[\frac{s^2+\frac{3}{2}s+\frac{3}{2}}{s(s+3)}\right]$$

$$\Rightarrow \Phi = s^2 + \frac{3}{2}s + \frac{3}{2}$$

Gain Checks-

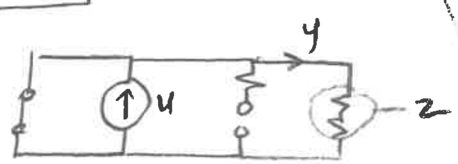
$\boxed{s=0}$ (Cap open, Ind shorted)



$y = \underbrace{\left(\frac{1}{1+2}\right)}_{\text{current division}} u = \frac{1}{3} u$

$\Rightarrow H(0) = \frac{1}{3}$

$\boxed{s=\infty}$ (Cap shorted, Ind open)



$\Rightarrow y = 0$

$\Rightarrow H(\infty) = 0$

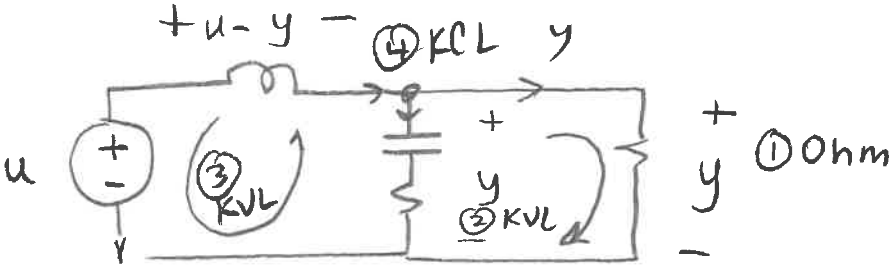
all agree with what we got earlier!

250	280

Problem # 8

Determine & plot $|H(j\omega)|$ & $\underline{/H(j\omega)}$

(all $R = L = C = 1$)



④ KCL =

$$\boxed{\left(\frac{u-y}{s}\right) = \left(\frac{y}{\frac{1}{s}+1}\right) + \left(y\right)}$$

$$\frac{1}{s}+1 = \frac{s+1}{s}$$

$$\frac{u}{s} = y\left[\frac{1}{s} + \frac{s}{s+1} + 1\right]$$

$$(s+1)u = y\left[s+1 + s^2 + s(s+1)\right]$$

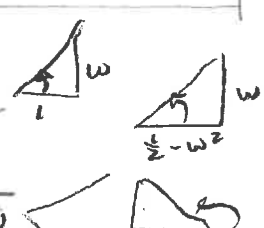
$$s(s+1) = s^2+s$$

$$= y\left[2s^2 + 2s + 1\right]$$

$$= y\,2\left[s^2 + s + \tfrac{1}{2}\right]$$

$$\boxed{y = \left[\frac{\frac{1}{2}(s+1)}{s^2+s+\frac{1}{2}}\right]u}$$

$\uparrow$ H



$\omega \in [0, \frac{1}{\sqrt{2}}]$

$\omega \in [\frac{1}{\sqrt{2}}, \infty)$

$$H(j\omega) = \frac{\frac{1}{2}(1+j\omega)}{\frac{1}{2}-\omega^2+j\omega}$$

$$|H(j\omega)| = \frac{\frac{1}{2}\sqrt{1+\omega^2}}{\sqrt{(\frac{1}{2}-\omega^2)^2+\omega^2}}$$

$$\underline{/H(j\omega)} = \tan^{-1}\left(\frac{\omega}{1}\right) - \begin{cases} \tan^{-1}\left(\frac{\omega}{\frac{1}{2}-\omega^2}\right) & \omega \in [0, \frac{1}{\sqrt{2}}] \\ 180 - \tan^{-1}\left(\frac{\omega}{\omega^2-\frac{1}{2}}\right) & \omega \in [\frac{1}{\sqrt{2}}, \infty) \end{cases}$$

260 | 280

Problem # 8

Note =

From $$H(s) = \frac{\frac{1}{2}(s+1)}{s^2+s+\frac{1}{2}}$$

$$H(0) = 1 = 1e^{j0^\circ}$$

$$H(s) \overset{s\,\text{large}}{\cong} \frac{\frac{1}{2}s}{s^2} = \frac{\frac{1}{2}}{s} \Rightarrow \begin{array}{l} |H(j\infty)| = 0 \\ \angle H(j\infty) = -90^\circ \end{array}$$

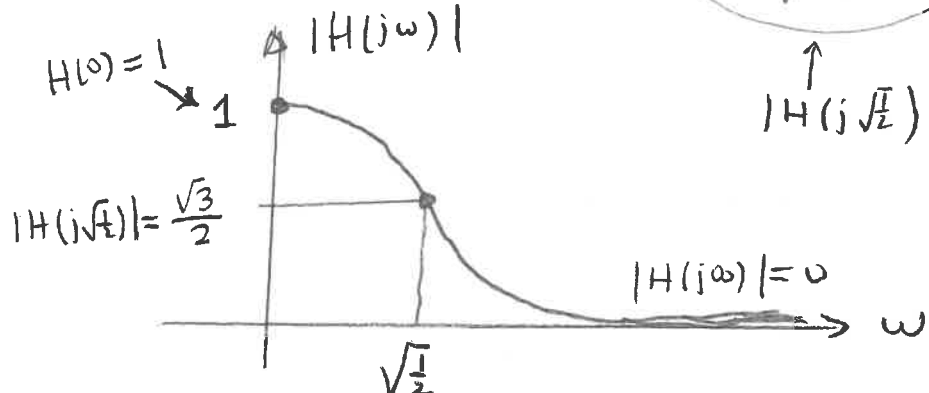
$$H\left(j\sqrt{\tfrac{1}{2}}\right) = \frac{\frac{1}{2}\left(1+j\sqrt{\frac{1}{2}}\right)}{-\frac{1}{2}+j\sqrt{\frac{1}{2}}+\frac{1}{2}}$$

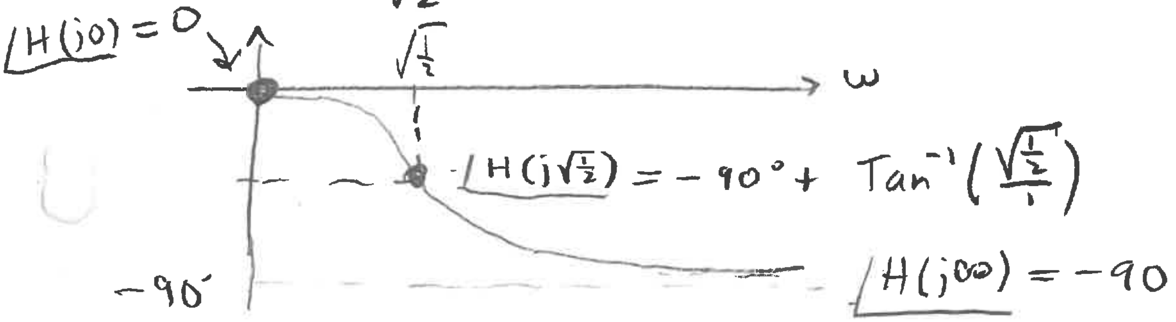
$$= \frac{\frac{1}{2}\left(1+j\sqrt{\frac{1}{2}}\right)}{j\sqrt{\frac{1}{2}}}$$

$$= \frac{\frac{1}{2}\sqrt{1+\frac{1}{2}}}{\sqrt{\frac{1}{2}}}\, e^{j\left(\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}}}{1}\right)-90^\circ\right)}$$

$\angle H(j\sqrt{\tfrac{1}{2}})$

$$\left|H\left(j\sqrt{\tfrac{1}{2}}\right)\right| = \tfrac{1}{2}\sqrt{2+1} = \frac{\sqrt{3}}{2}$$

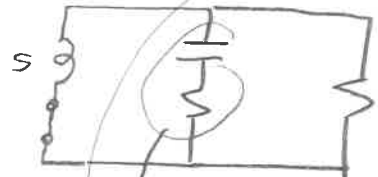




$\boxed{270 | 280}$

Problem #8

Φ Check: Set $u=0$



$\frac{1}{s}+1=\frac{s+1}{s}$

$\left(\frac{s+1}{s}\right)\|1=\dfrac{\frac{s+1}{s}}{\frac{s+1}{s}+1}$

$=\frac{s+1}{2s+1}$

$\Longrightarrow Z = s + \frac{s+1}{2s+1}$

$= \frac{2s^2+s+s+1}{2s+1}$

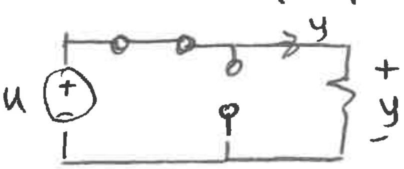
$= \frac{2s^2+2s+1}{2s+1}$

$= 2\left[\frac{s^2+s+\frac{1}{2}}{2s+1}\right]$

$\Longrightarrow \Phi = s^2+s+\frac{1}{2}$

Gain Checks:

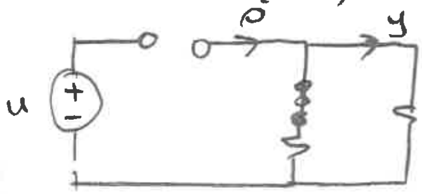
$\boxed{s=0}$ (Ind shorted, cap open)



$y=u$

$H(0)=1$

$\boxed{s=\infty}$ (Ind open, cap open)



$\Rightarrow y=0$

$\Rightarrow H(\infty)=0$

All these agree with

$H=\dfrac{\frac{1}{2}(s+1)}{s^2+s+\frac{1}{2}}$

obtained earlier!

$\boxed{280|280}$