EXAMS & EXAM SOLUTIONS

FALL 2023

Dr. A.A. Rodriguez

EE202

Exam Rules

(As discussed in class & "formally announced")

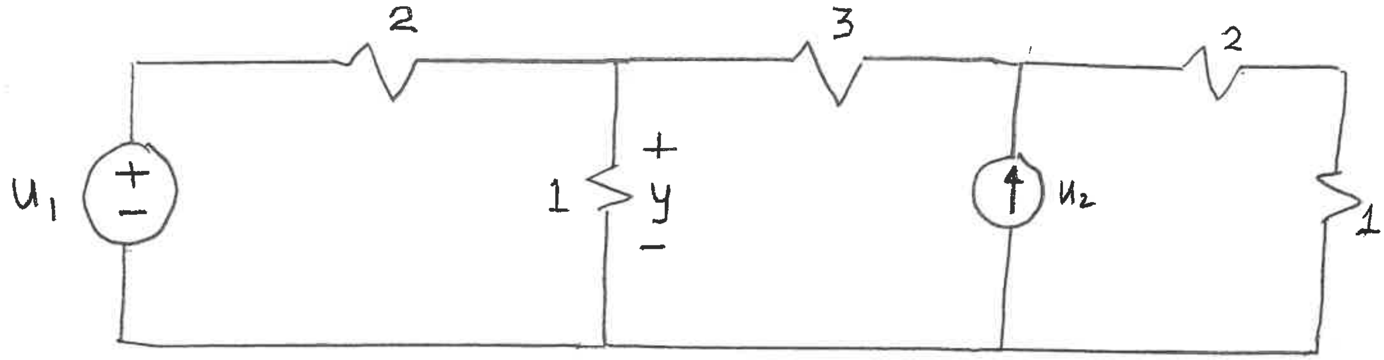
Exam # 1
Fall 2023
10-25-23

1. One $8\frac{1}{2}'' \times 11''$ sheet permitted; otherwise closed books, closed notes, closed phones, open minds!
2. Calculator permitted (no computer!!; no phone!!!)
3. <u>NO PHONES</u>! Phones should not be visible at all! A visible phone will result in a zero grade for the exam!
4. Write full name legibly on each page provided.
5. Please show all work. Write your solutions on the pages provided. (No other pages should be used!)
6. Clearly label voltages and currents on circuits provided.
7. Use variables provided! (No additional variables!)
8. Unreadable work will receive no credit.
9. Please place important equations and answers within boxes.
10. Please turn in solutions to me at end of period!
11. PLEASE DO NOT CHEAT!!!

[redacted]

Problem # 1

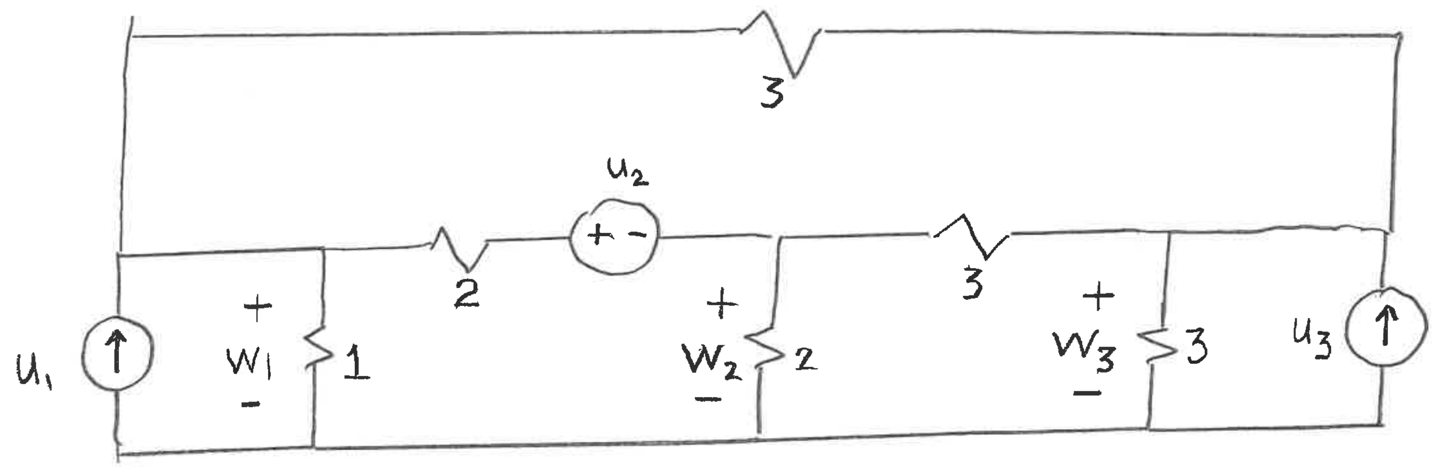
Relate y to u_1, u_2 (Propagate y)



Problem #2

Relate w_1, w_2, w_3 to u_1, u_2, u_3

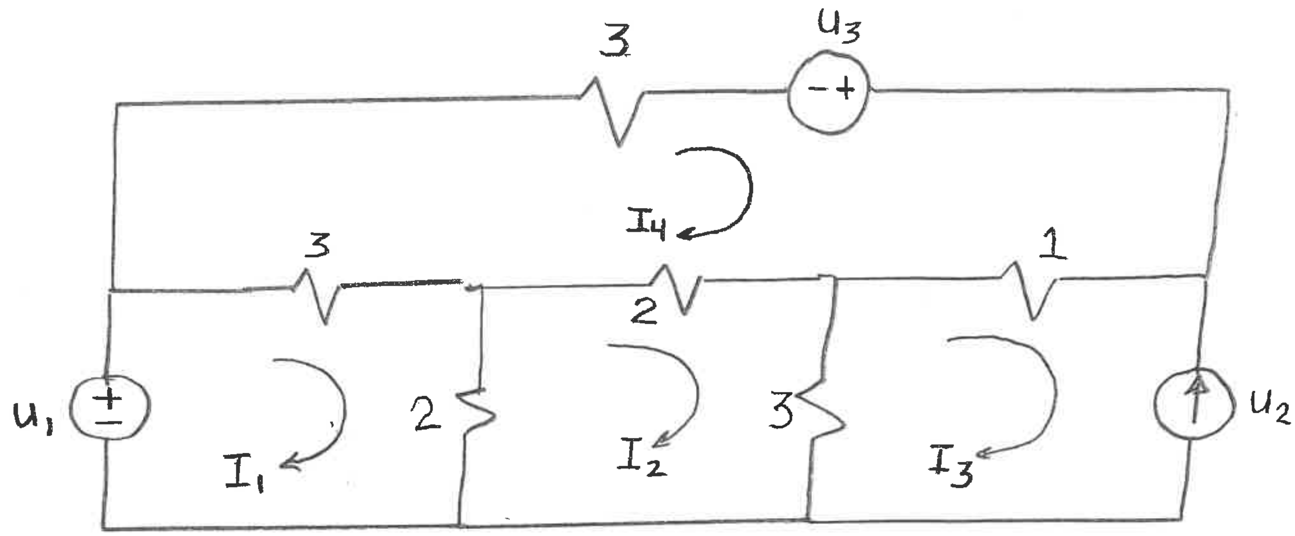
(Nodal Analysis)



Problem # 3

Relate I_1, I_2, I_3, I_4 to u_1, u_2, u_3

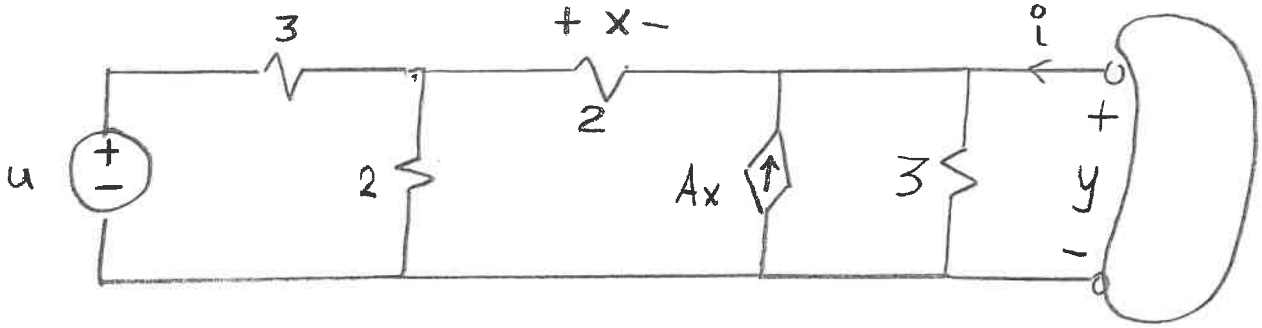
(Mesh Analysis)



Problem # 4

Find a Thevenin equivalent at y
(looking leftward)

(Thevenin Equivalent at y)

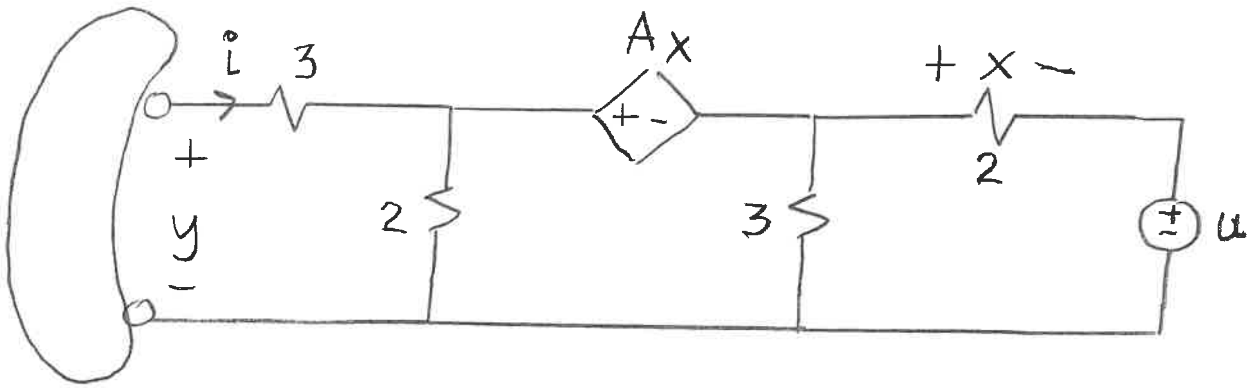


Problem # 5

Find a Thevenin equivalent at y

(looking rightward)

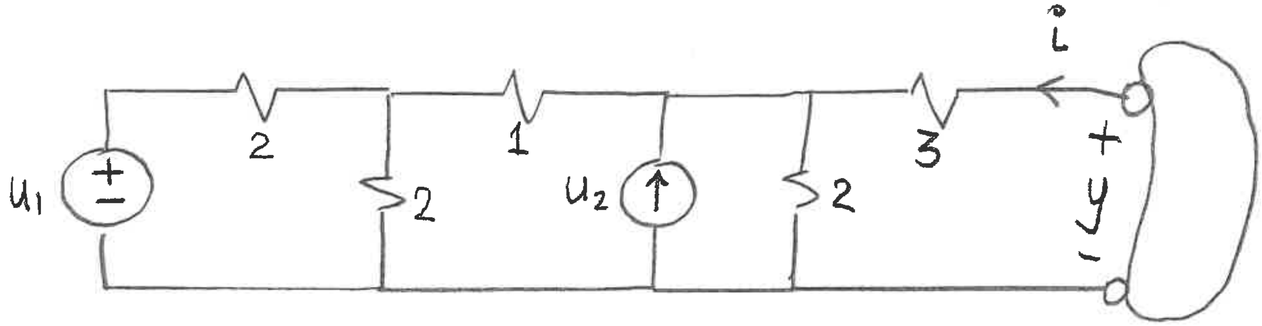
(Thevenin Equivalent at y)



Problem #6

Find a Thevenin Equivalent at y
(looking leftward)

(Thevenin Equivalent at y)



Exam Rules

Exam #2
Fall 2023
Fri 12-8-23

Dr. A.A. Rodriguez
EE202

1. One $8\frac{1}{2}$" x 11" sheet permitted; otherwise closed books, closed notes, closed phones, open minds!

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6. Clearly label voltage & currents on circuits provided.

7. Use variables provided! (No additional variables!)

8. Unreadable work will receive NO CREDIT.

9. Please place important equations & answers within box.

10. Please be careful with your algebra, signs, etc.

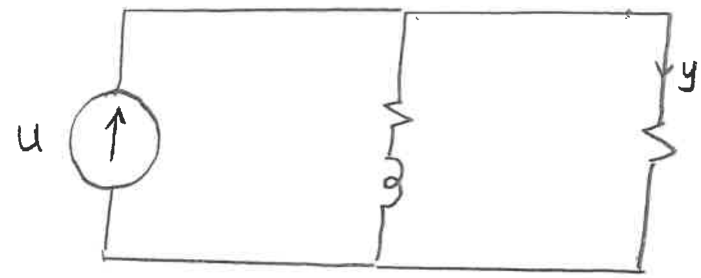
11. Please turn in solutions to me at end of period!

12. PLEASE DO NOT CHEAT!!!

Problem # 1

Determine H, diffeq, τ, t_s, y_{ss}, y $\quad u = -3 + \cos 2t$

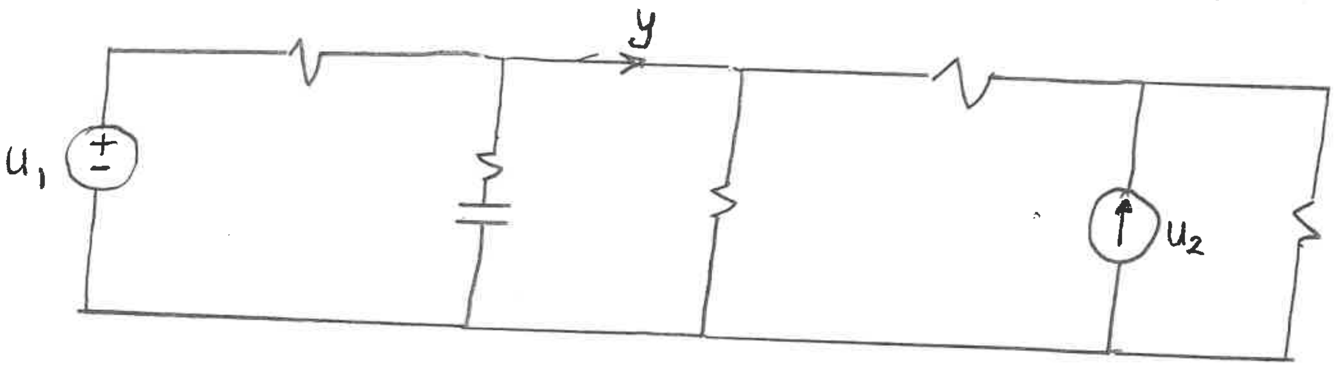
(all $R = L = C = 1$)



Problem #2

Determine $H_1(0)$, $H_2(0)$, $H_1(\infty)$, $H_2(\infty)$, τ, t_s

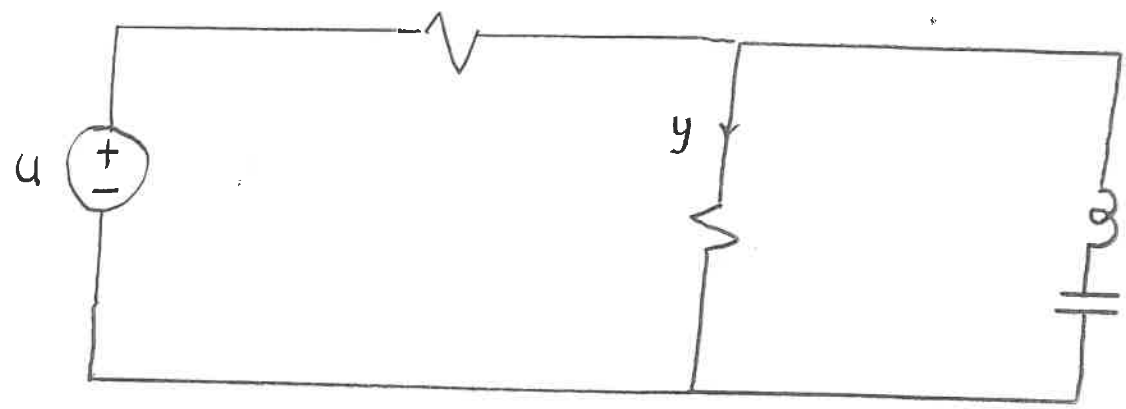
(all R=L=C=1)



Problem # 3

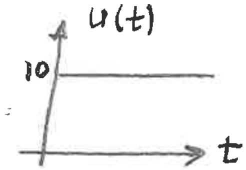
Determine H, diffeq, T, ts, yss $u = -5 + 10\sin(t+3($

(all R = L = C = 1)

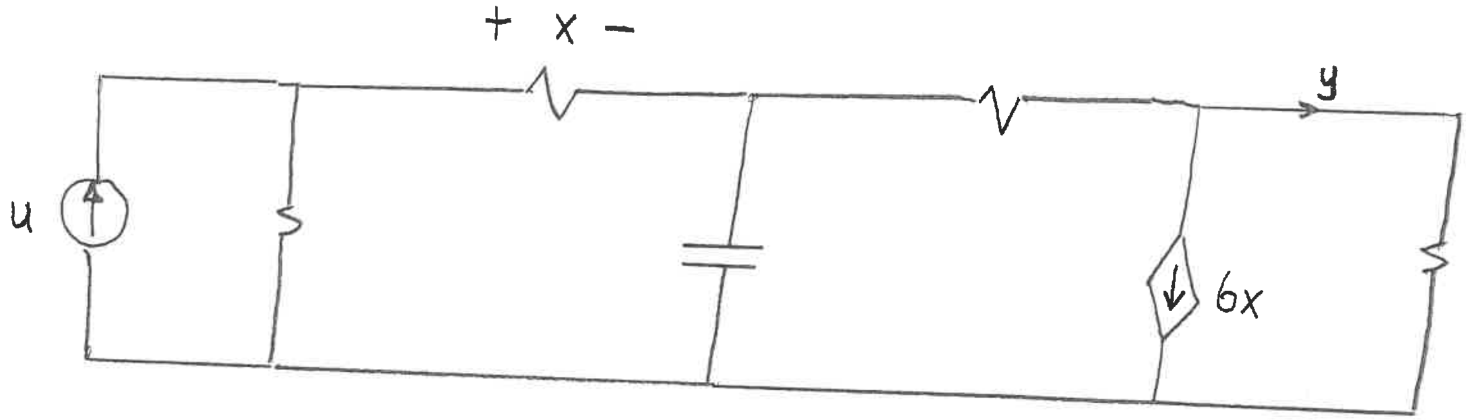


Problem # 4

Determine H, diffeq, y



(all $R=L=C=1$)



Problem #5

Determine y_{1ss}, y_1, t_{s1}, y_{2ss}, y_2, t_{s2}

$$H_1(s) = \frac{s}{(s+2)(s^2-6s+25)}$$

$$u_1(t) = -10 + 50\cos(0.01t - 45^\circ) + 8\sin(5t + 30^\circ)$$

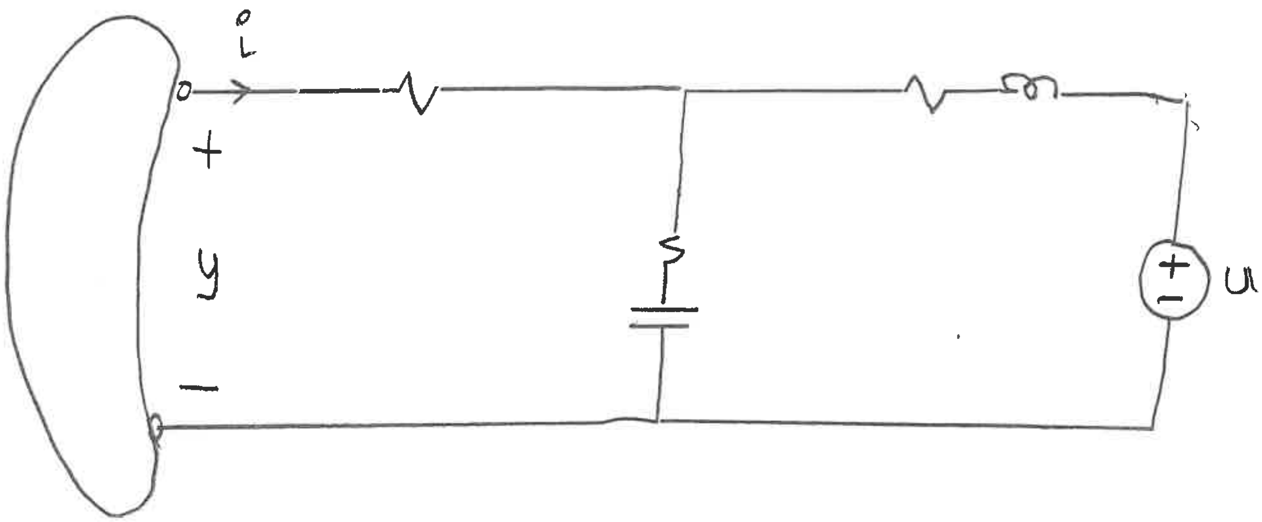
$$H_2(s) = \frac{s^2+1}{(s+1)(s^2+10s+100)}$$

$$u_2(t) = 7 - \cos(t + 20^\circ) + 6\sin(100t + 135^\circ)$$

Problem #6

Determine an s-domain Thevenin Equivalent at y

(all R = L = C = 1)



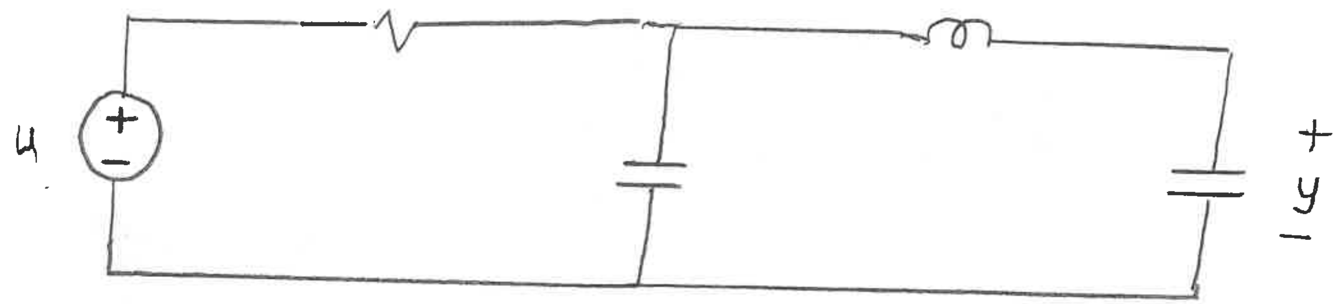
Problem # 7

Determine H & y_{ss}

$u = -5 + 2\sin(t+30$

$-3\cos(100t - 135°)$

(all $R = L = C = 1$)



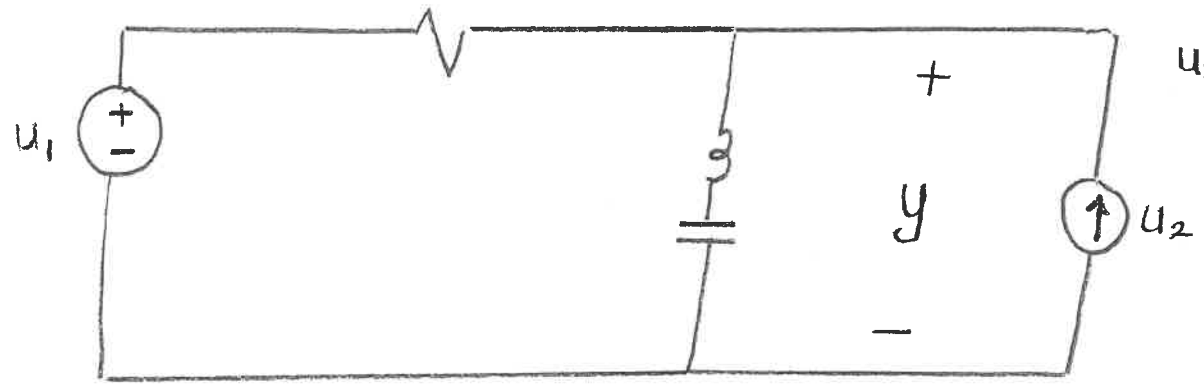
Problem # 8

Determine H_1, H_2, t_{s1}, y_{1ss}

$u_1 = 10 - 8\sin(t+30^\circ) + 7\cos(100t+45^\circ)$

$u_2 = 0$

(all $R=L=C=1$)



Dr. A.A. Rodriguez
EE202

Exam Rules

Final Exam
Fall 2023
Fri 12-15-23

1. One $8\frac{1}{2}'' \times 11''$ sheet permitted; otherwise closed books, closed notes, closed phones; open minds!
2. Calculator permitted (no computer!!; no phone!!!)
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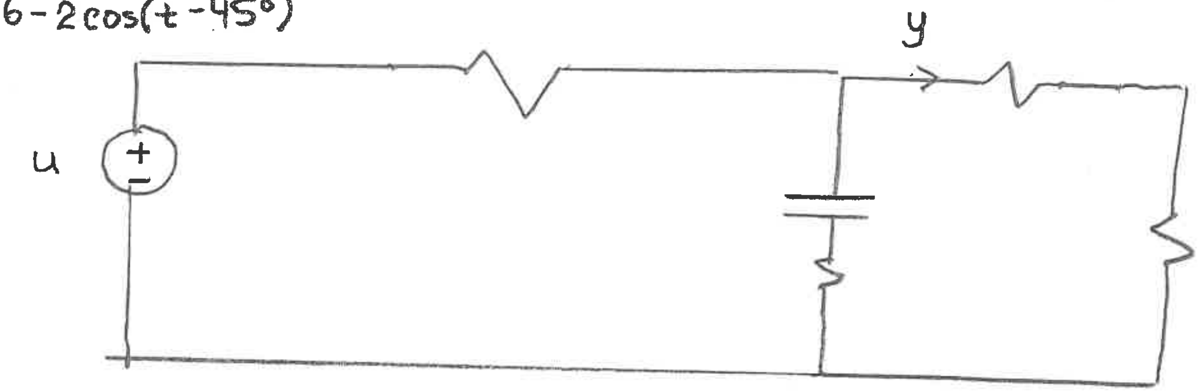
Problem #1

Determine H, diff eq, t_s, y_{ss}, y

(all R=L=C=1)

$u = -6 - 2\cos(t - 45°)$

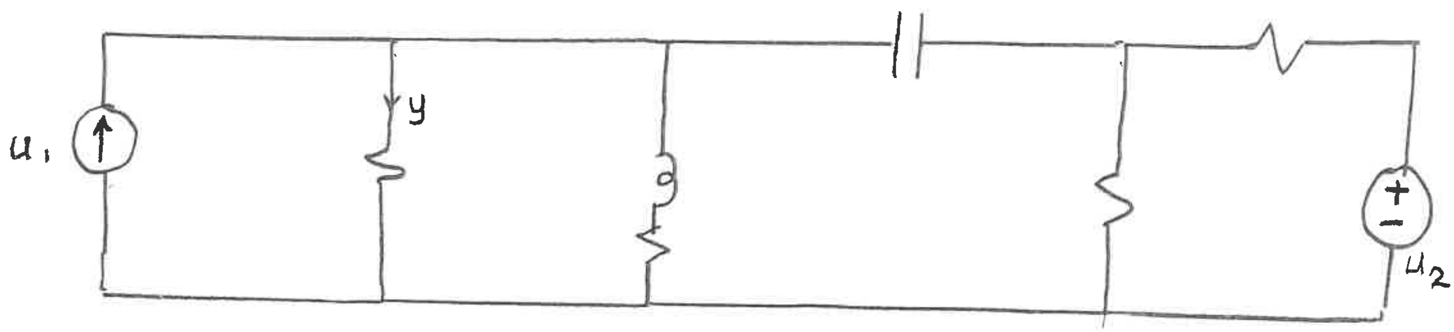


Final Fall 2023

Problem # 2

Determine $H_1(0)$, $H_2(0)$, $H_1(\infty)$, $H_2(\infty)$, poles, t_s

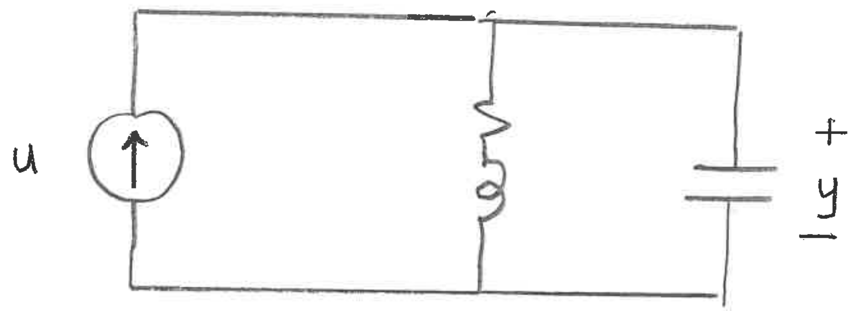
(all $R=L=C=1$)



Problem # 3

Determine H, diff eq, ts, y_{ss}

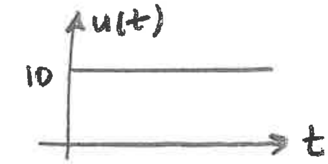
(all R = L = C = 1)



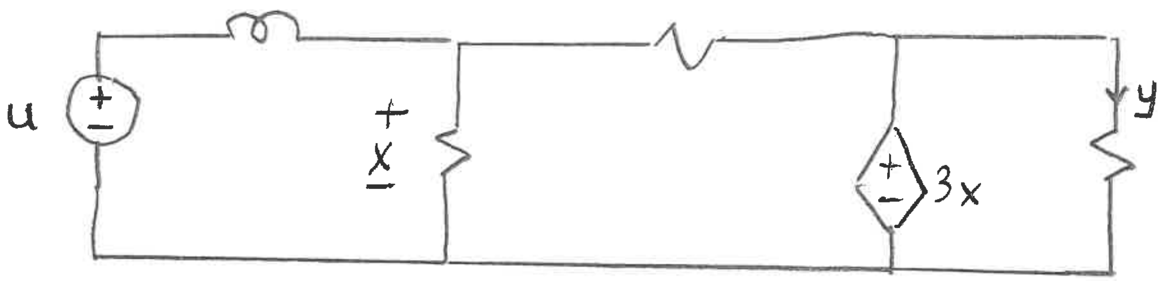
$u = 4 - 3\sin(t + 135°)$

Problem # 4

Determine H, diff eq, y



(all R=L=C=1)



Problem # 5

Determine y_{ss}, y, t_s

$$H(s) = \frac{s(s^2+4)}{(s+2)(s^2+2s+10)(s^2-s+1)}$$

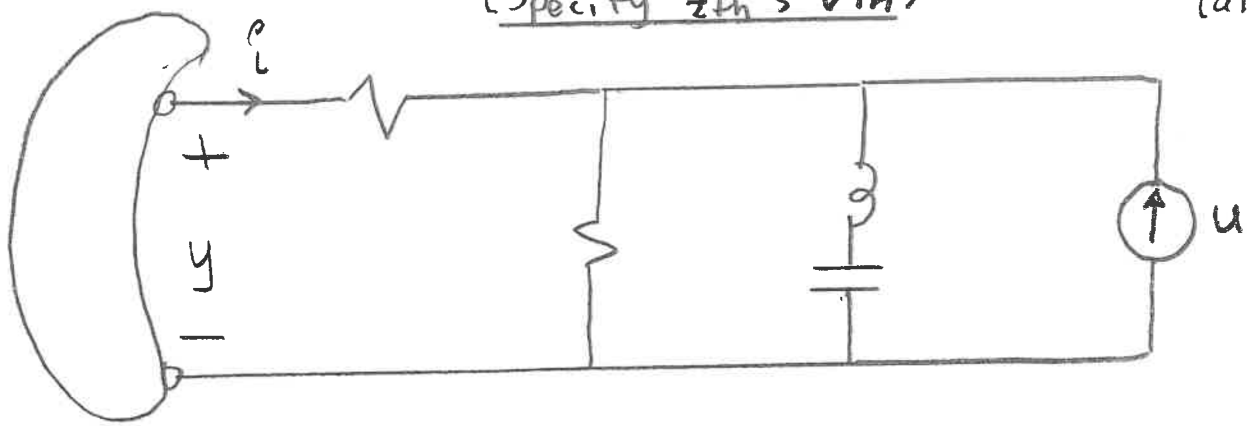
$u(t) = 5 - 6\sin(2t + 45°) + 7\cos(100t + 270°)$

(compute all important coefficients as discussed in class)

Problem #6

Determine an s-domain Thevenin Equivalent at y (Specify Z_{th} & V_{th})

(all R=L=C=1)

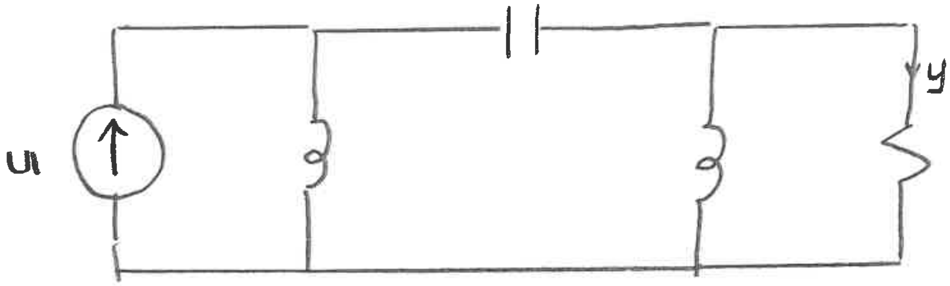


Problem # 7

Determine H & y_{ss}

[Compute all coefficients in y_{ss} (approximately)]

(all $R=L=C=1$)



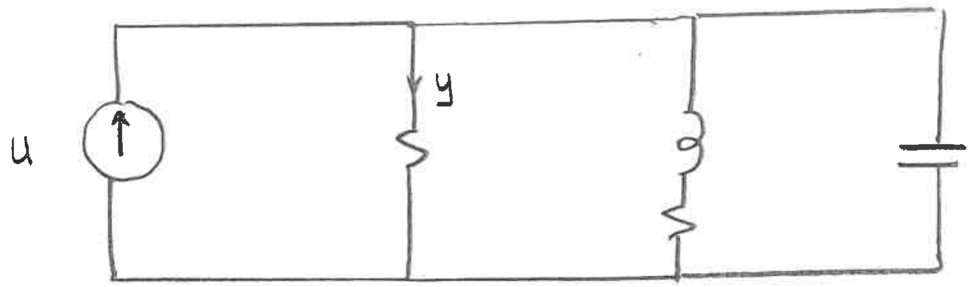
$$u = -7 + 8\cos(0.01t - 180°) - 9\sin(100t + 20°)$$

Problem #8

Determine H, t_s, y_{ss}

$u = -8 + 9\sin(t - 45°)$

(all R=L=C=1)



Exam #1 Rules

(EEE 202)

Wed 10-25-23
Exam 1
Fall 2023 Solutions

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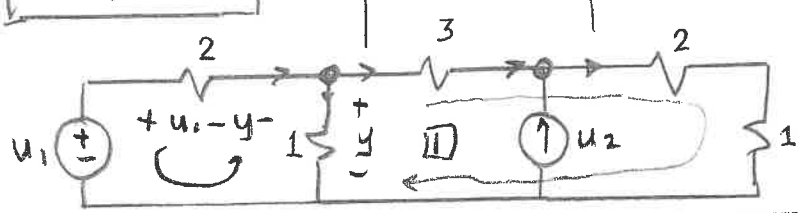
AAR Solutions
Exam #1 Fall 2023

Problem 1

(Propagate y)

$\left[\frac{u_1-y}{2}\right]-y$ $\qquad$ $\left[\frac{u_1-y}{2}\right]-y+u_2$



Relate y to $u_{1,2}$

$$\boxed{1}\ \text{KVL} = \boxed{y = 3\left[\frac{u_1-y}{2}\right] - 3y + \overset{2+1}{3}\left\{\left[\frac{u_1-y}{2}\right]-y+u_2\right\}}$$

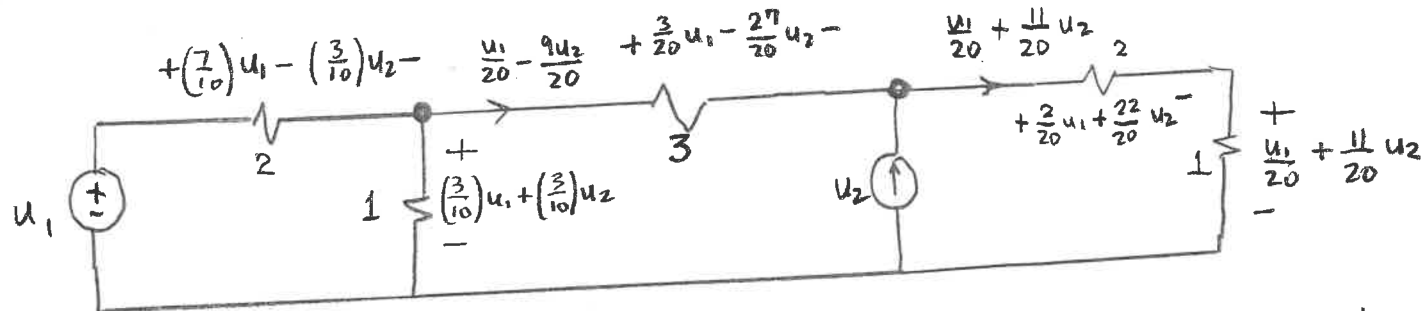
$$y\left[\underset{\frac{2}{2}}{1} + \frac{3}{2} + \underset{\frac{6}{2}}{3} + \frac{3}{2} + \underset{\frac{6}{2}}{3}\right] = u_1\left[\frac{3}{2} + \frac{3}{2}\right] + 3u_2$$

$$y\left[\frac{20}{\cancel{2}}\right] = \left(\frac{6}{\cancel{2}}\right)u_1 + \left(\frac{6}{\cancel{2}}\right)u_2 \implies \boxed{y = \left(\frac{3}{10}\right)u_1 + \left(\frac{3}{10}\right)u_2}$$

$$\frac{u_1}{2} - \frac{3}{2}y = \overset{\left(\frac{10}{20}\right)u_1}{\frac{u_1}{2}} - \frac{9}{20}u_1 - \frac{9}{20}u$$

$$= \left(\frac{1}{20}\right)u_1 - \left(\frac{9}{20}\right)u_2$$

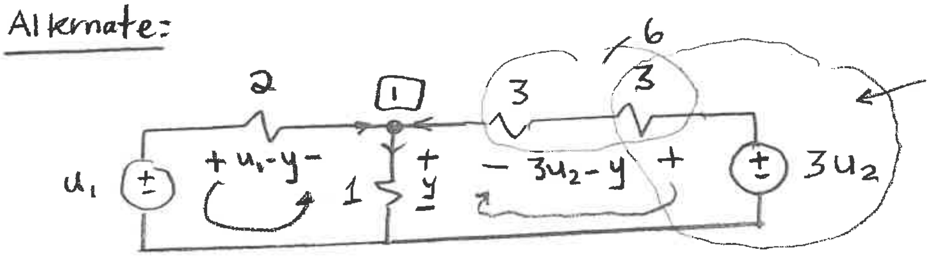
Check:



KVL & KCL check out everywhere!

Alternate:

i to v
Source Transformation

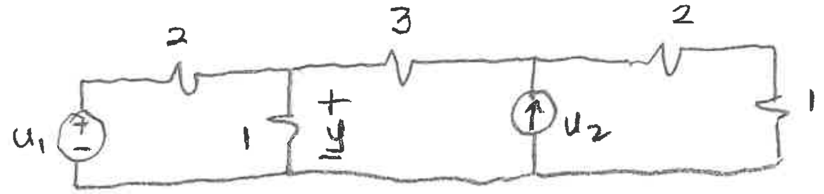


$$\boxed{1}\ \text{KCL} = \boxed{\left[\frac{u_1-y}{2}\right] + \left[\frac{3u_2-y}{6}\right] = \left[\frac{y}{1}\right]}$$

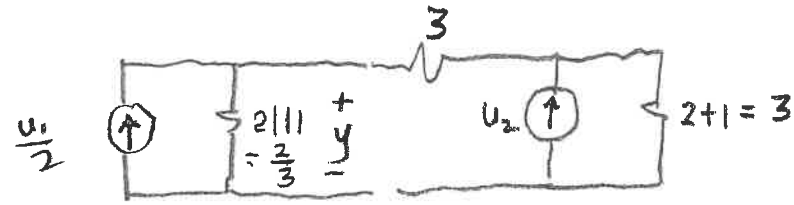
6x $\qquad 3u_1 - 3y + 3u_2 - y = 6y$

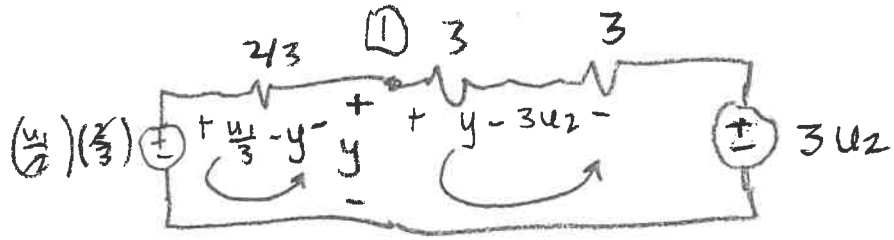
$$10y = 3u_1 + 3u_2$$

$$\implies \boxed{y = \left(\frac{3}{10}\right)u_1 + \left(\frac{3}{10}\right)u_2}\ \checkmark$$



<u>sol via source transf</u>





① KCL =
$$\left(\frac{\frac{u_1}{3} - y}{\left(\frac{2}{3}\right)}\right) = \left(\frac{y - 3u_2}{3+3}\right) \quad 6$$

$$\frac{u_1}{2} - \frac{3}{2}y = \frac{y}{6} - \frac{1}{2}u_2$$

$$3u_1 - 9y = y - 3u_2$$

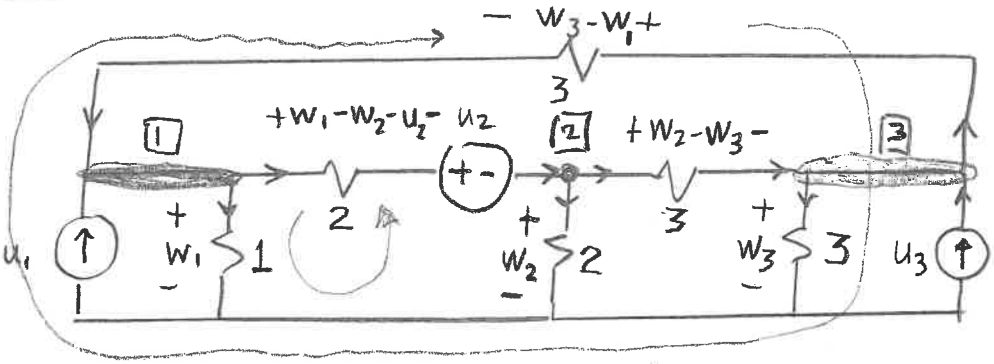
$$10y = 3u_1 + 3u_2$$

$$\boxed{y = \left(\frac{3}{10}\right)u_1 + \left(\frac{3}{10}\right)u_2}$$ ✓

Problem # 2

Relate $w_{1,2,3}$ to $u_{1,2,3}$

(Nodal Analysis)



$\boxed{1}$ KCL =

$$\boxed{\left(\frac{W_3-W_1}{3}\right)+u_1=\left(\frac{W_1}{1}\right)+\left(\frac{W_1-W_2-U_2}{2}\right)}$$

$\boxed{2}$ KCL =

$$\boxed{\left(\frac{W_1-W_2-U_2}{2}\right)=\left(\frac{W_2}{2}\right)+\left(\frac{W_2-W_3}{3}\right)}$$

$\boxed{3}$ KCL

$$\boxed{\left(\frac{W_2-W_3}{3}\right)+\left(u_3\right)=\left(\frac{W_3}{3}\right)+\left(\frac{W_3-W_1}{3}\right)}$$

Matrix-Vector Form:

$$\underbrace{\begin{bmatrix} 1+\frac{1}{2}+\frac{1}{3} & -\frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & -\frac{1}{2}-\frac{1}{2}-\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{b_1} u_1 + \underbrace{\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix}}_{b_2} u_2 + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{b_3} u_3$$

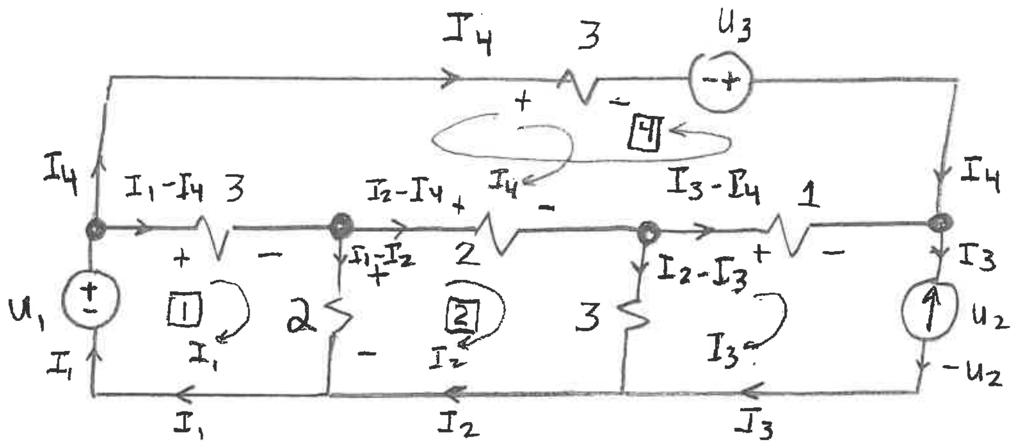
If x_i solves $Ax_i = b_i$ then by linearity:

$$X = \begin{bmatrix} | \\ x_1 \\ | \end{bmatrix} u_1 + \begin{bmatrix} | \\ x_2 \\ | \end{bmatrix} u_2 + \begin{bmatrix} | \\ x_3 \\ | \end{bmatrix} u_3$$

Problem # 3

Relate I_1, I_2, I_3, I_4 to u_1, u_2, u_3

(Mesh Analysis)



1 KVL = $\boxed{u_1 = 3(I_1 - I_4) + 2(I_1 - I_2)}$

2 KVL = $\boxed{2(I_1 - I_2) = 2(I_2 - I_4) + 3(I_2 - I_3)}$

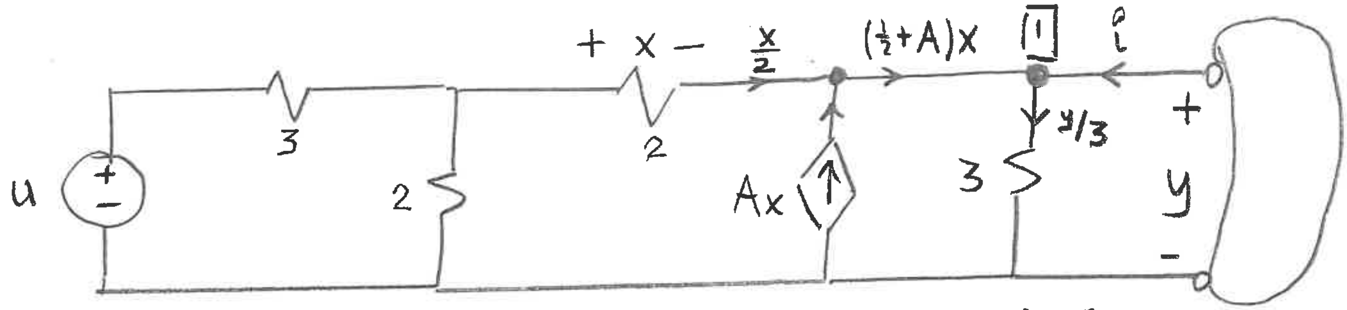
3 KCL = $\boxed{I_3 = -u_2}$

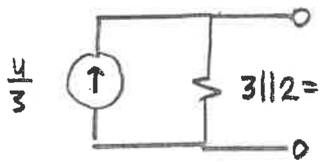
4 KVL = $\boxed{3(I_4) - u_3 = 3(I_1 - I_4) + 2(I_2 - I_4) + 1(I_3 - I_4)}$

Problem # 4

Find a Thevenin Equivalent at y (looking leftward)

(Thevenin Equivalent at y)

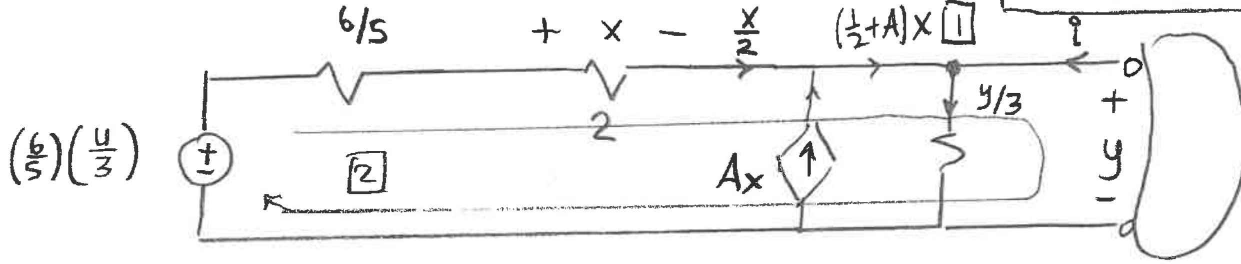




$3||2 = \frac{(3)(2)}{3+2} = \frac{6}{5}$

[1] KCL =

$$\boxed{\left(\tfrac{1}{2}+A\right)x + i = \left(\tfrac{y}{3}\right)}$$



[2] KVL =

$$\boxed{\underbrace{\left(\tfrac{6}{5}\right)\left(\tfrac{u}{3}\right)}_{(2/5)u} = \left[\tfrac{6}{5} + \underbrace{2}_{10/5}\right]\left[\tfrac{x}{2}\right] + y}$$

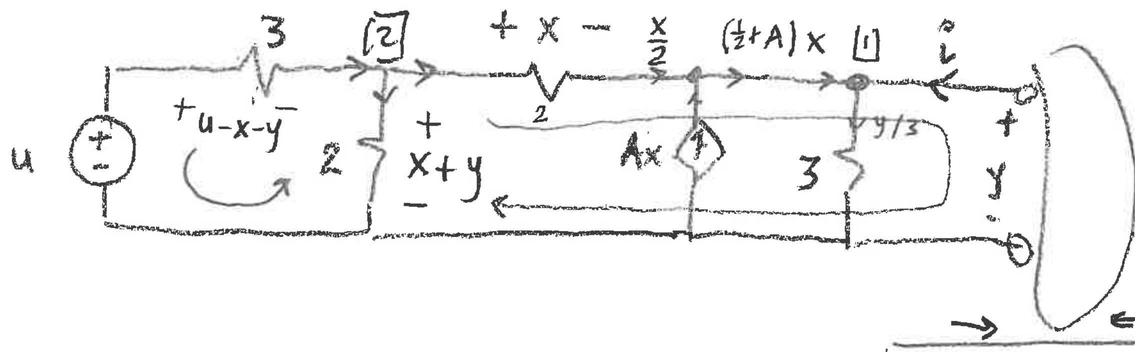
[1] $\Rightarrow \quad x = \left[\frac{y/3 - i}{A+\frac{1}{2}}\right]$ $\quad \overset{[2]}{\Rightarrow} \quad \left(\frac{2}{5}\right)u = \left(\frac{\cancel{16}^{\,8}}{5}\right)\frac{1}{\cancel{2}}\left[\frac{y/3 - i}{A+\frac{1}{2}}\right] + y$

$$y\left[\frac{\frac{8/5}{3}}{A+\frac{1}{2}} + 1\right] = \left[\frac{\frac{8}{5}}{A+\frac{1}{2}}\right]i + \left(\frac{2}{5}\right)u$$

$$y\left[\frac{8}{15} + A + \frac{1}{2}\right] = \left[\frac{8}{5}\right]i + \left(\frac{2}{5}\right)\left(A+\frac{1}{2}\right)u$$

$$y\left[\underbrace{8 + 15A + 7.5}_{15.5}\right] = 24i + 6\left(A+\frac{1}{2}\right)u$$

$$\boxed{y = \underbrace{\left[\frac{24}{15A+15.5}\right]}_{R_{th}} i + \underbrace{6\left[\frac{A+\frac{1}{2}}{15A+15.5}\right]}_{V_{th}} u}$$



① KCL: $\boxed{\left(\tfrac{1}{2}+A\right)x + i = y/3}$

$$\Downarrow$$

$$x = \left(\frac{y/3 - i}{A+\frac{1}{2}}\right)$$

② KCL: $\boxed{\left(\frac{u-x-y}{3}\right) = \left(\frac{x+y}{2}\right) + \left(\frac{x}{2}\right)}$

$$\frac{u}{3} = x\left[\frac{1}{3}+\frac{1}{2}+\frac{1}{2}\right] + y\left[\frac{1}{3}+\frac{1}{2}\right]$$

$6\times$

$$2u = x\,[\underbrace{2+3+3}_{8}] + y\,[\underbrace{2+3}_{5}]$$

$$\Rightarrow \quad 5y + 8x = 2u$$

$$5y + 8\left[\frac{y/3 - i}{A+\frac{1}{2}}\right] = 2u$$

$$5\left(A+\tfrac{1}{2}\right)y + \frac{8}{3}y - 8i = 2\left(A+\tfrac{1}{2}\right)u$$

$$15\left(A+\tfrac{1}{2}\right)y + 8y - 24i = 6\left(A+\tfrac{1}{2}\right)u$$

$$y\,[15A + 7.5 + 8] = 24i + 6\left(A+\tfrac{1}{2}\right)u$$

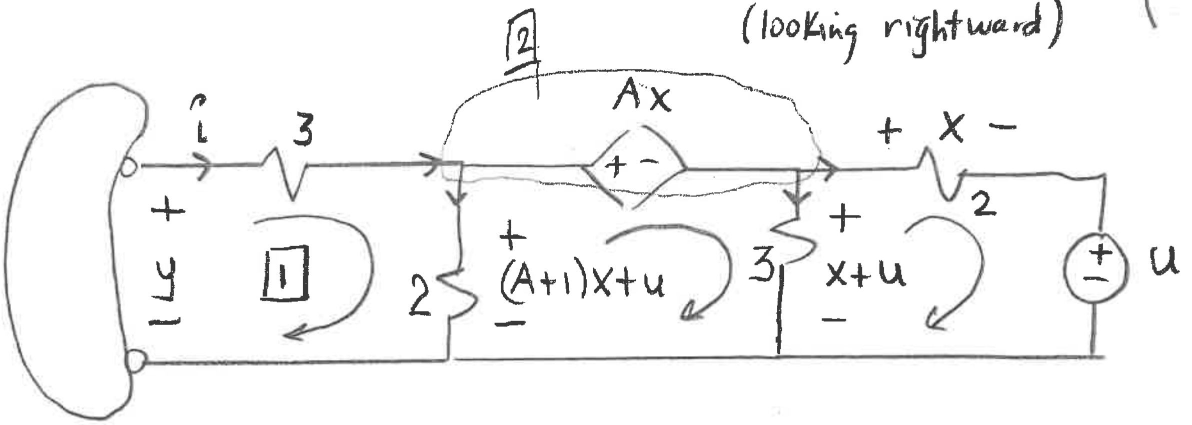
$$\boxed{y = \underbrace{\left(\frac{24}{15A+15.5}\right)}_{R_{th}} i + \underbrace{6\left[\frac{A+\frac{1}{2}}{15A+15.5}\right]u}_{V_{th}}}$$

Problem#5

Find a Thevenin Equivalent at y

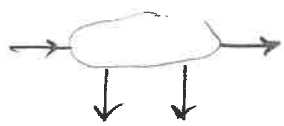
(Thevenin Equivalent at y)

(looking rightward)



$\boxed{1}$ KVL= $\boxed{y = 3i + (A+1)x + u}$

$\boxed{2}$ KCL=



$$\boxed{i = \left[\frac{(A+1)x+u}{2}\right] + \left[\frac{x+u}{3}\right] + \left[\frac{x}{2}\right]}$$

6x

$$6i = 3(A+1)x + 3u + 2x + 2u + 3x$$

$$= x[3A + \underbrace{8}_{3+2+3}] + u[\underbrace{3+2}_{5}]$$

$$\Longrightarrow \quad x = \frac{6i-5u}{3A+8}$$

$$\Longrightarrow \quad y = 3i + (A+1)\underbrace{\left[\frac{6i-5u}{3A+8}\right]}_{x} + u$$

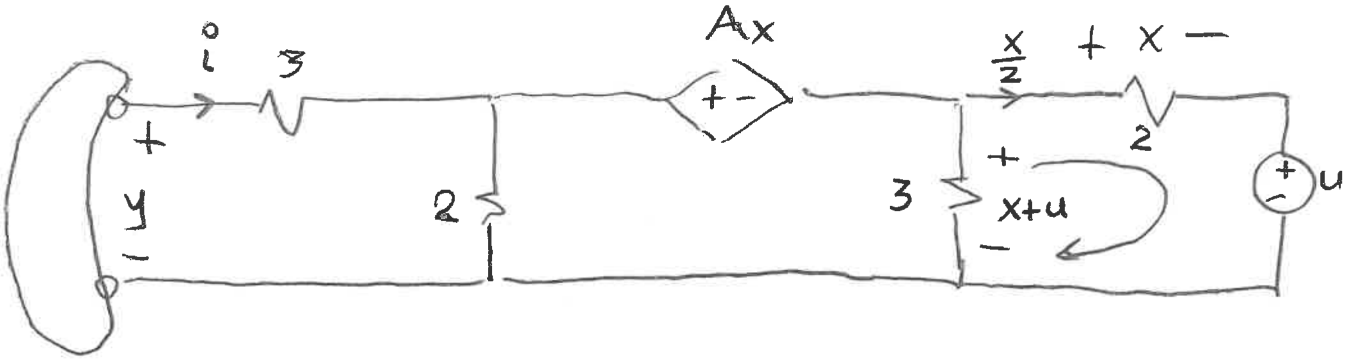
$$\Longrightarrow \quad (3A+8)y = i[3(3A+8) + (A+1)6] + u[(3A+8)$$

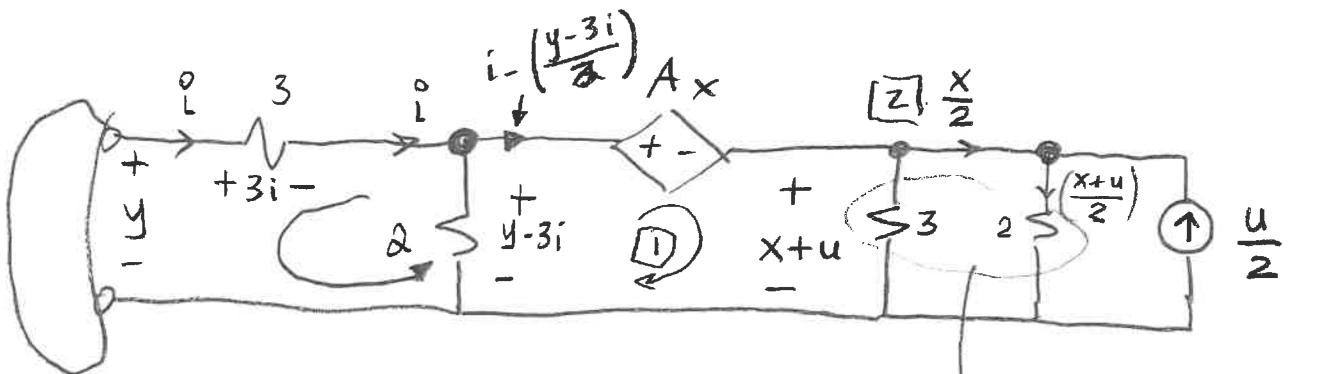
$$= i[9A + 24 + 6A + 6] + u[3A+8 \underbrace{-5A-5}_{-5(A+1)}]$$

$$= i[15A+30] + u[3-2A]$$

$$\boxed{y = \underbrace{\left[15\left(\frac{A+2}{3A+8}\right)\right]}_{R_{th}} i + \underbrace{\left(\frac{3-2A}{3A+8}\right)}_{V_{th}} u}$$

solution via source transf





$3 \| 2 = \frac{(3)(2)}{3+2} = \frac{6}{5} = 1.2$

1 KVL = $\boxed{(y-3i) = (Ax) + (x+u)}$

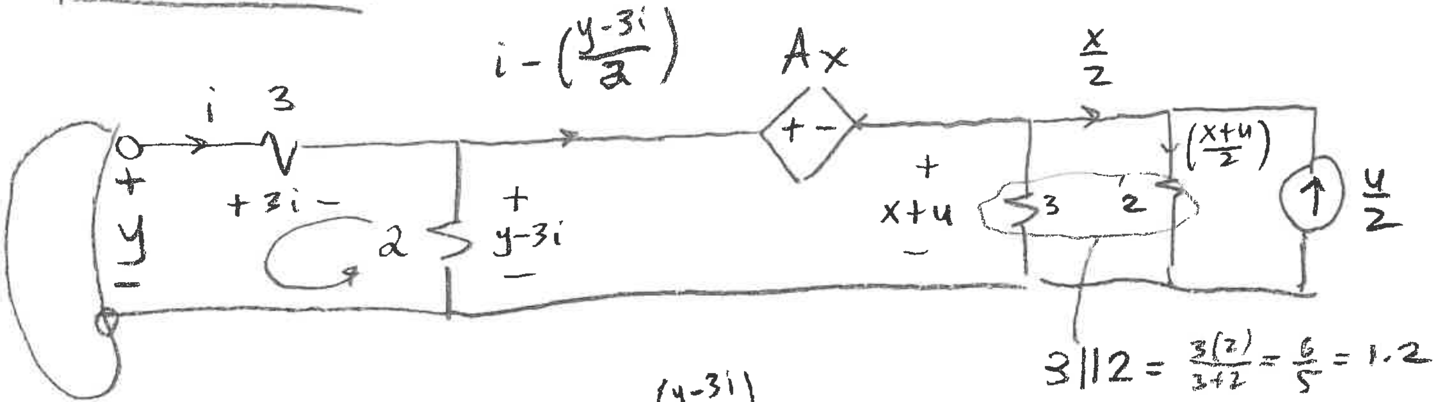
or $y = 3i + (A+1)x + u$ as on reverse side

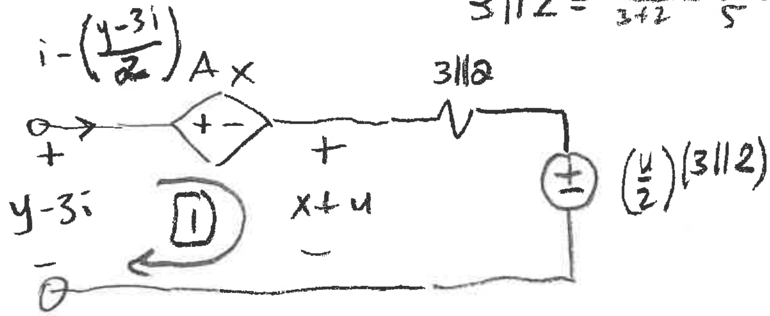
Also, KCL at 2 yields $\boxed{i - \left(\frac{y-3i}{2}\right) = \left(\frac{x+u}{3}\right) + \left(\frac{x}{2}\right)}$

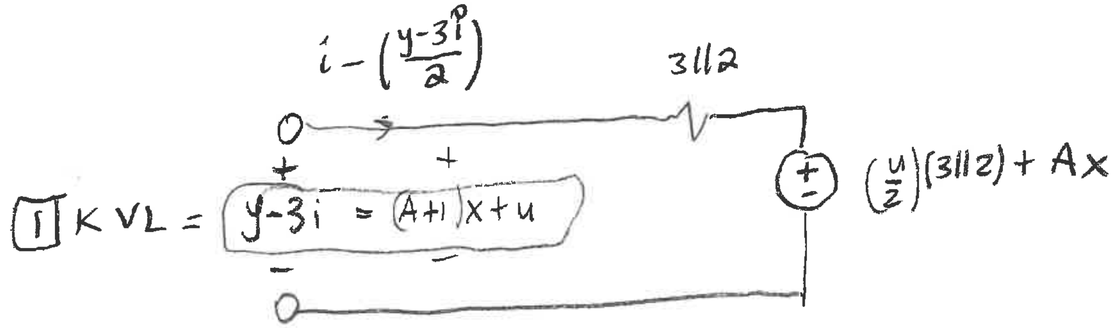
(same as on reverse side!)

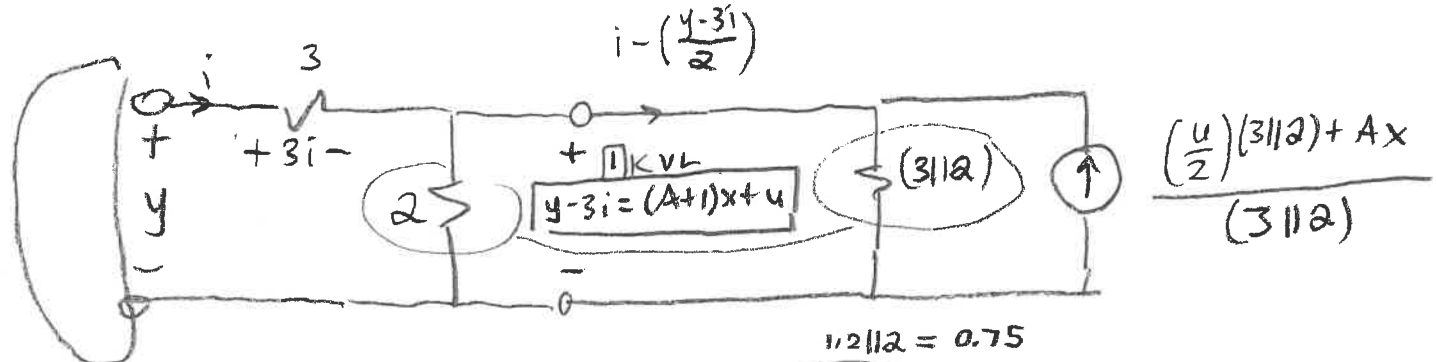
Problem #5

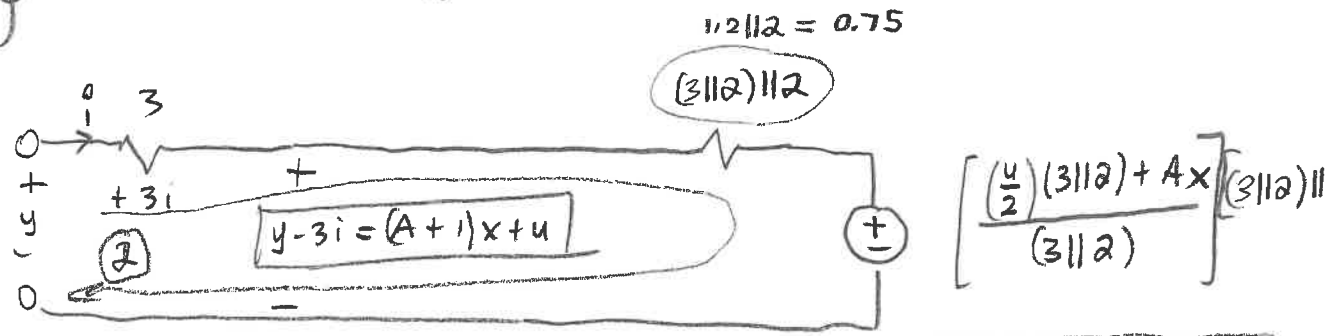
(sol via Source transf)











$\boxed{2}$ KVL = $y = \left(3 + (3\|2)\|2\right) i + \left[\dfrac{\left(\frac{u}{2}\right)(3\|2) + Ax}{(3\|2)}\right]\left[(3\|2)\|2\right]$

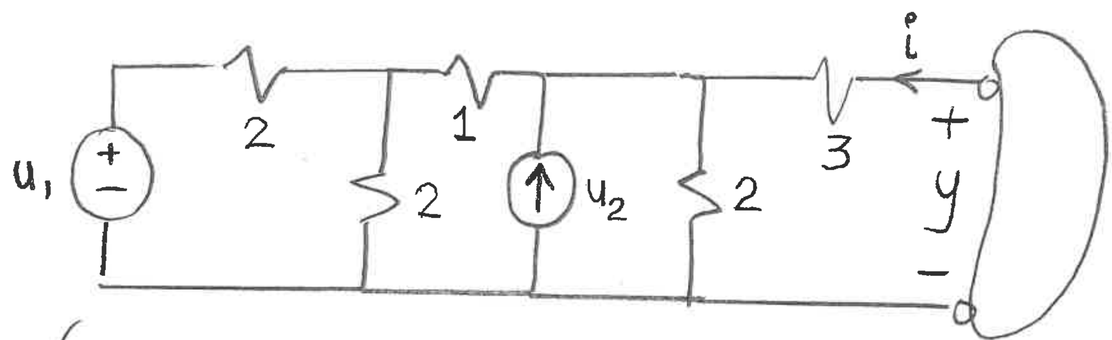
- We have 2 eq in 2 unknowns y, x !!!

Problem #6

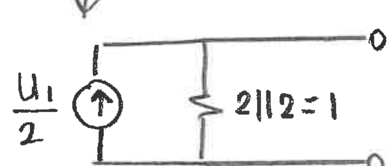
Find a Thevenin Equivalent at y

(looking leftward)

(Thevenin Equivalent at y)



Source Transf



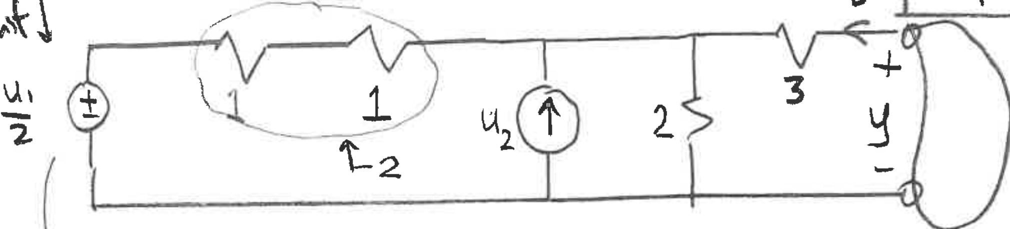
Note: $Req = 3 + \left[2 \,\|\, [1 + (2\|2)]\right] = 3 + 2\|(1+1)$

$= 3 + (2\|2)$

$= 3 + 1$

$\boxed{Req = 4}$ (best way to find R_{th}!!!)

Source Transf



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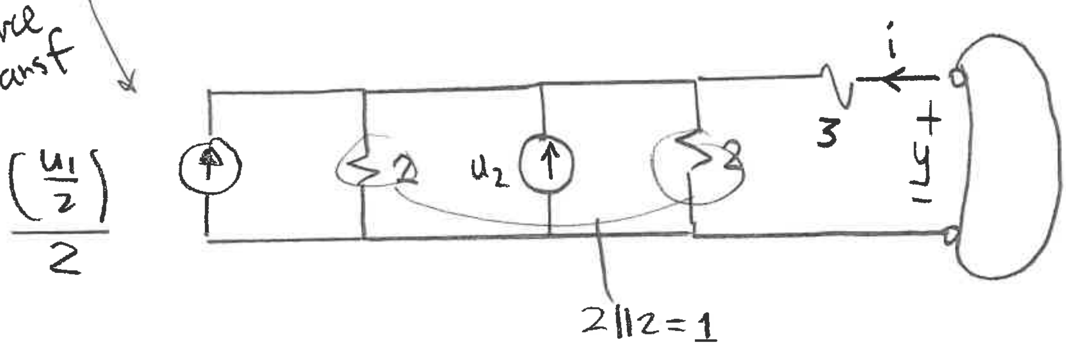

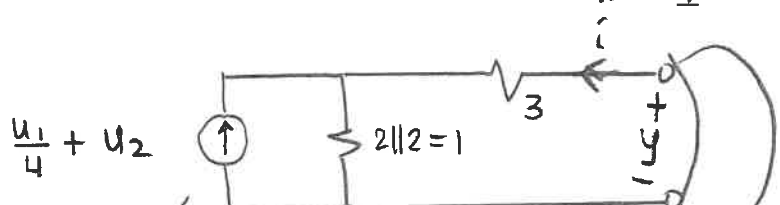

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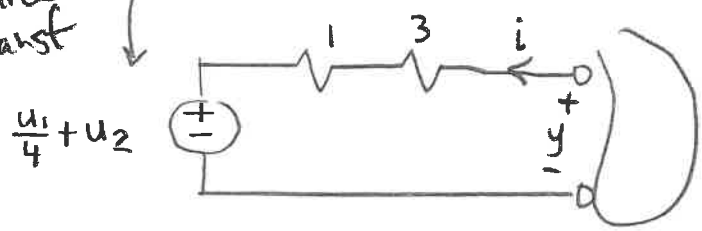
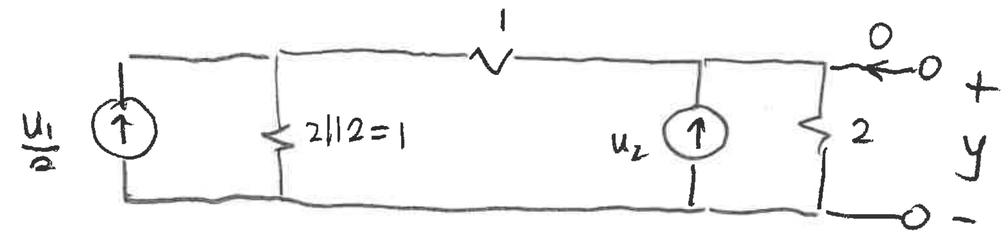
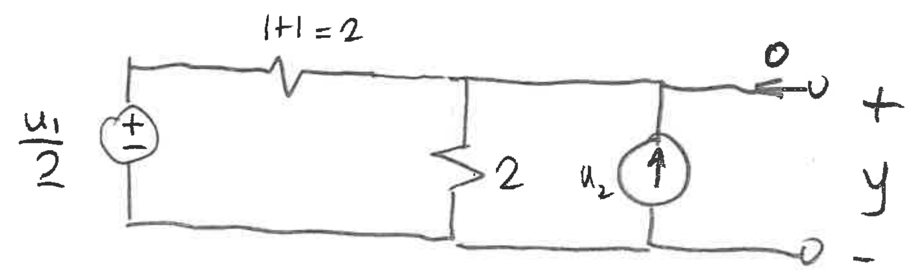
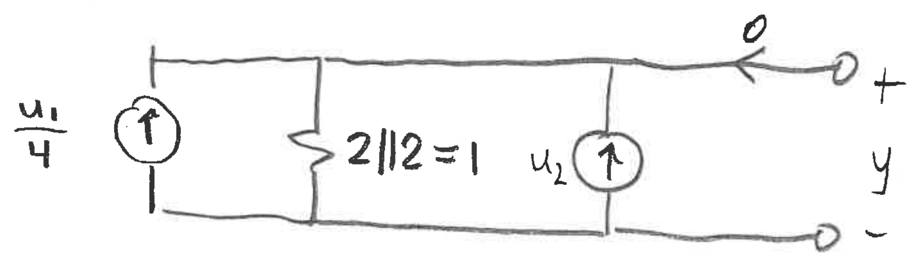


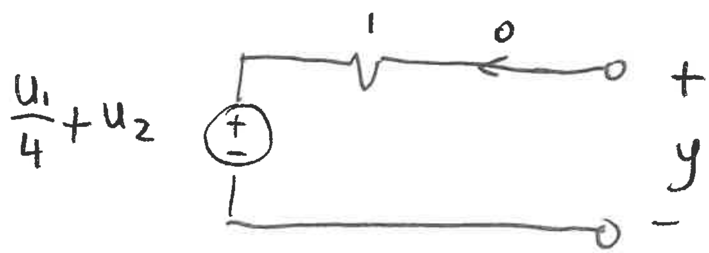
$$\boxed{\begin{array}{l} R_{th} = 4 \\ V_{th} = \frac{u_1}{4} + u_2 \end{array}}$$

Check on $V_{th} = y|_{i=0}$ (Source transf)







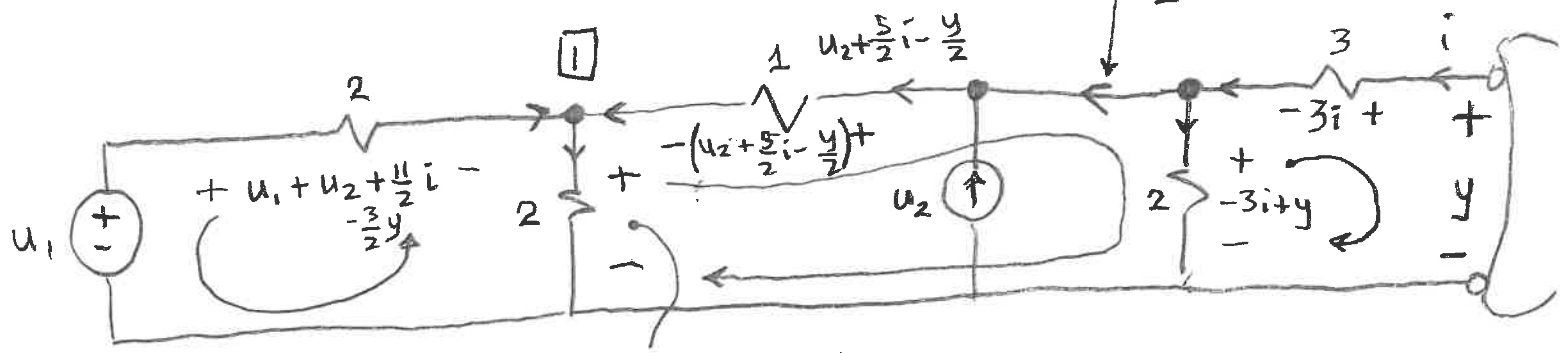


$$y|_{i=0} = V_{th} = \frac{u_1}{4} + u_2 \checkmark$$

Problem 6

(Thev equiv = sol via KVL, KCL, etc.)

$$i - \left(\frac{-3i+y}{2}\right) = \frac{5}{2}i - \frac{y}{2}$$



$$-u_2 - \frac{5}{2}i + \frac{y}{2} - 3i + y$$
$$= -\frac{11}{2}i + \frac{3}{2}y - u_2$$

$$\boxed{1}\ \text{KCL} = \boxed{\left(\frac{u_1 + u_2 + \frac{11}{2}i - \frac{3}{2}y}{2}\right) + \left(\frac{u_2 + \frac{5}{2}i - \frac{y}{2}}{1}\right) = \left(\frac{-\frac{11}{2}i + \frac{3}{2}y - u_2}{2}\right)}$$

Algebra:

$$y\left[\frac{3}{4} + \overset{2/4}{\frac{1}{2}} + \frac{3}{4}\right] = i\left[\frac{11}{4} + \overset{10/4}{\frac{5}{2}} + \frac{11}{4}\right] + u_1\left[\frac{1}{2}\right] + u_2\left[\frac{1}{2} + 1 + \frac{1}{2}\right]$$

$$y\left[\underset{2}{\frac{8}{4}}\right] = i\left[\underset{8}{\frac{32}{4}}\right] + \left[\frac{1}{2}u_1 + 2u_2\right]$$

$$\boxed{y = (4)i + \left[\frac{1}{4}u_1 + u_2\right]}$$

$\boxed{R_{th} = 4}$ ✓

$\boxed{V_{th} = \frac{1}{4}u_1 + u_2}$

Exam Rules

Fri 12-8-23
Fall 2023

AAR Exam #2
solutions

1. One $8\frac{1}{2}$" x 11" sheet permitted; otherwise closed books, closed notes, closed phone open minds!
2. Calculator permitted (no computer!!; no phone!!!)
3. NO PHONES! Phones should not be visible at all! A visible phone will result in a zero grac for the exam!
4. Write full name legibly on each page provided.
5. Please show all work. Write your solutions on the pages provided. (No other pages/paper should be used!)
6. Clearly label voltage & currents on circuits provided.
7. Use variables provided! (No additional variables!)
8. Unreadable work will receive NO CREDIT.
9. Please place important equations & answers within box
10. Please be careful with your algebra, signs, etc.
11. Please turn in solutions to me at end of period!
12. PLEASE DO NOT CHEAT!!! ☹

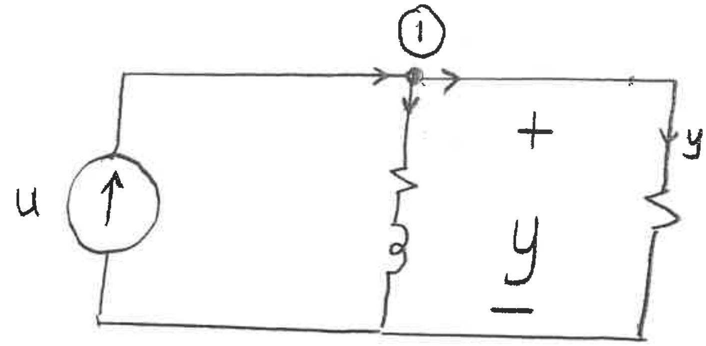
Dr. A.A. Rodriguez
EE 202

AAR SOLUTIONS
EXAM #2 Fall 2023

Problem # 1

Determine H, diffeq, τ, t_s, y_{ss}, y $\qquad u = -3 + \cos 2t$

(all $R = L = C = 1$)



$$\text{① KCL} = \boxed{u = \frac{y}{s+1} + y} = y\left[\frac{1}{s+1} + 1\right]$$

$$(s+1)u = y[1 + s + 1] = y[s+2]$$

$$\boxed{H = \frac{y}{u} = \frac{s+1}{s+2}} \qquad \boxed{\dot{y} + 2y = \dot{u} + u}$$

pole $= -2 \implies \tau = \frac{1}{2}$

(stable system) $\qquad t_s = 5\tau = \frac{5}{2}$ $\qquad \boxed{\tau = \frac{1}{2},\ t_s = \frac{5}{2}}$

MOTF

$$\boxed{y_{ss} = -3H(0) + |H(j2)| \cos(2t + \angle H(j2))}$$

$H(0) = \frac{1}{2}$

$$H(j2) = \frac{j2+1}{j2+2}$$

$\sqrt{5}$, 2, 1 $\qquad$ $2\sqrt{2}$, 45°, 2, 2

$$= \frac{\sqrt{5}\, e^{j \tan^{-1}(\frac{2}{1})}}{2\sqrt{2}\, e^{j45°}}$$

$$= \underbrace{\frac{\sqrt{5}}{2\sqrt{2}}}_{|H(j2)|} e^{j\overbrace{(\tan^{-1}(\frac{2}{1}) - 45)}^{\angle H(j2)}}$$

$$U = -\frac{3}{s} + \frac{s}{s^2+4} = \frac{\quad}{s(s^2+4)}$$

$$Y = HU = \left[\frac{s+1}{s+2}\right]\left[\frac{\quad}{s(s^2+4)}\right] \quad \pm j2$$

$$= \frac{A}{s} + \frac{B}{s+2} + \left[\frac{C}{s-j2} + *\right]$$

$$y(t) = A + Be^{-2t} + 2|C|\cos(2t + \angle C)$$

$$y_{ss} = A + 2|C|\cos(2t + \angle C)$$

$$= -3H(0) + |H(j2)|\cos(2t + \angle H(j2))$$

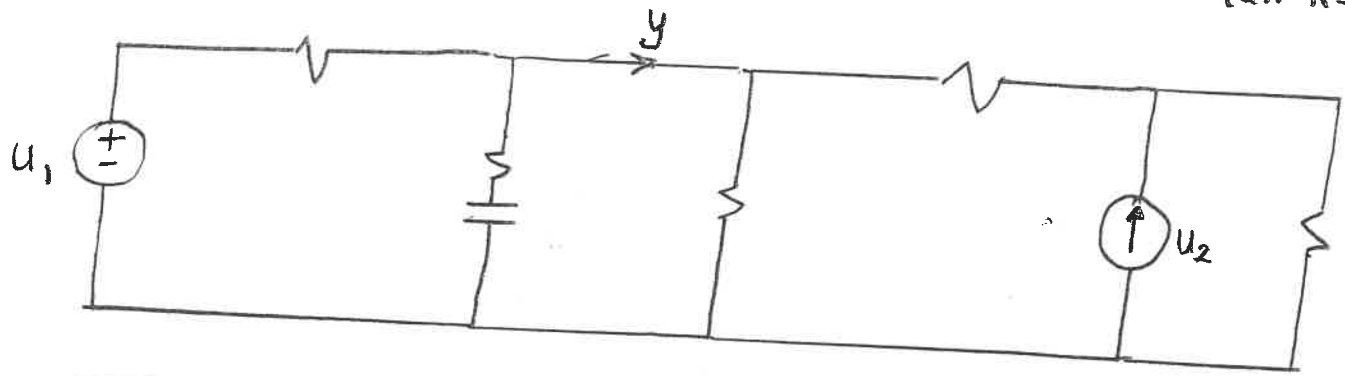
$$\boxed{A = \lim_{s \to 0} sY = -3H(0)} \qquad \boxed{C = \lim_{s \to j2} (s - j2)Y = \frac{|H(j2)|}{2} e^{j\angle H(j2)}}$$

$$B = \lim_{s \to -2} (s+2)Y$$

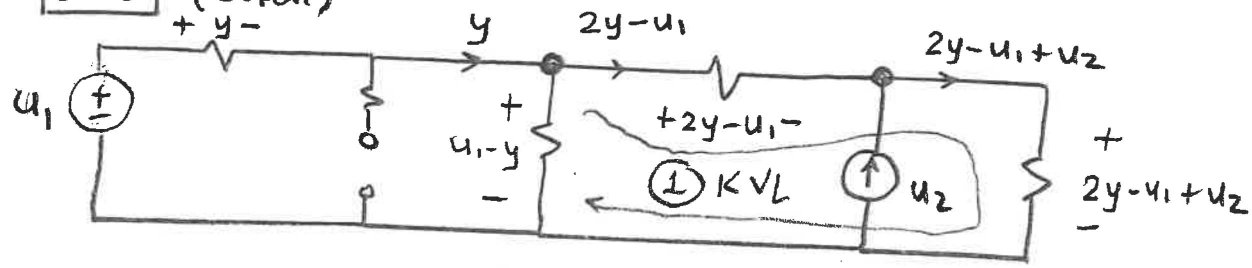
Problem #2

Determine $H_1(0)$, $H_2(0)$, $H_1(\infty)$, $H_2(\infty)$, τ.

(all $R=L=C=1$)



$s=0$ (C open)



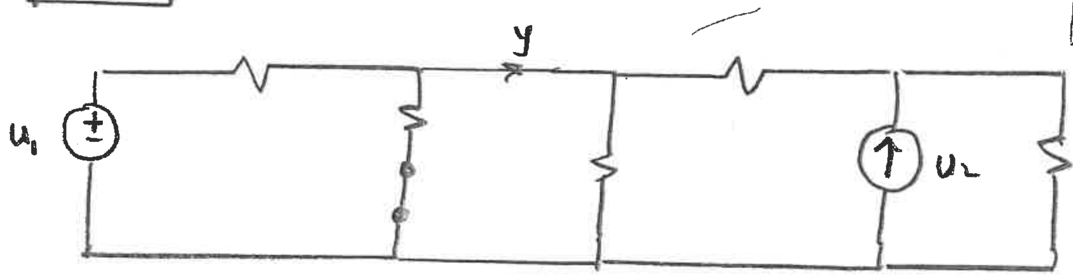
① KVL $=\quad u_1 - y = (2y-u_1) + (2y-u_1+u_2)$

$$3u_1 - u_2 = 5y$$

$$\boxed{y = \left(\tfrac{3}{5}\right)u_1 + \left(\tfrac{-1}{5}\right)u_2}$$

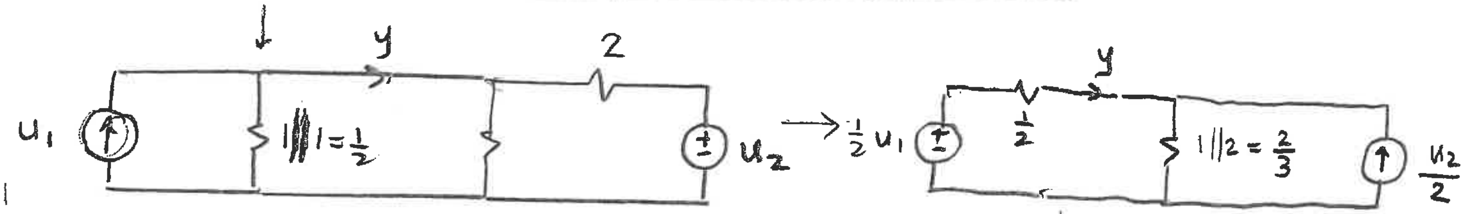
$H_1(0)$ $\quad$ $H_2(0)$

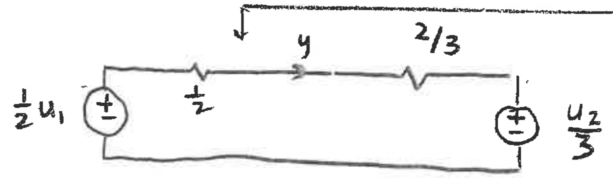
$s=\infty$



$$\boxed{y = \left(\tfrac{3}{7}\right)u_1 + \left(\tfrac{-2}{7}\right)u_2}$$

$H_1(\infty)$ $\quad$ $H_2(\infty)$





$\frac{3+4}{6} = \frac{7}{6}$

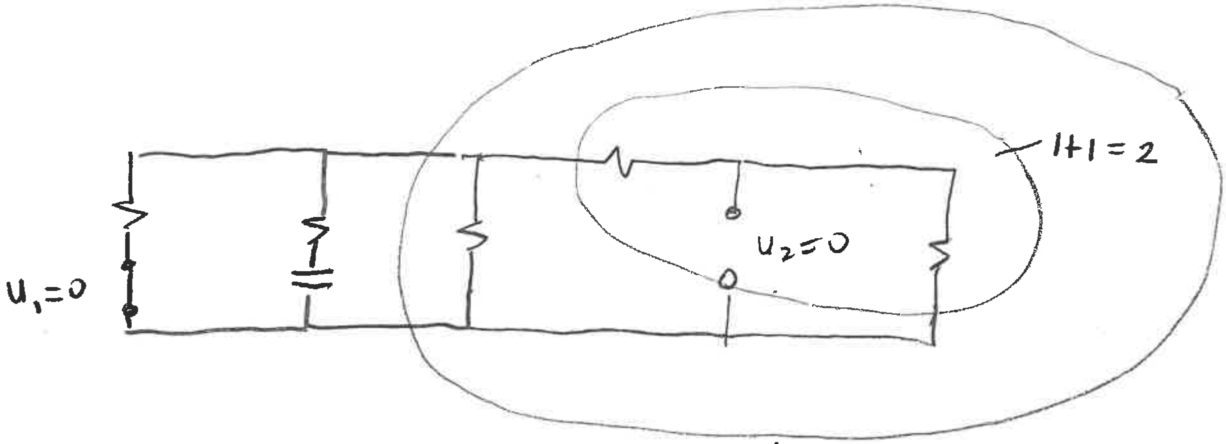
$\Longrightarrow$ KVL $=\ \frac{1}{2}u_1 = \left(\frac{1}{2}+\frac{2}{3}\right) y + \frac{u_2}{3}$

$$y = \left(\tfrac{\cancel{6}^{\,3}}{7}\right)\left(\tfrac{1}{\cancel{2}}\right)u_1 - \left(\tfrac{\cancel{6}^{\,2}}{7}\right)\left(\tfrac{1}{\cancel{3}}\right)u_2$$

$\boxed{3|9}$

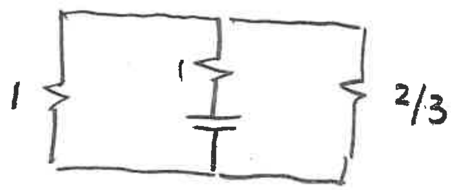
$\tau, t_s = ?$

Set $u_1 = u_2 = 0$



$1+1=2$

$1 \| 2 = \frac{(1)(2)}{1+2} = \frac{2}{3}$



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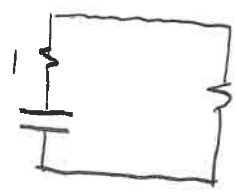
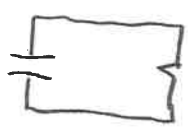


$1 \| \frac{2}{3} = \frac{(1)(\frac{2}{3})}{1 + \frac{2}{3}} = \frac{2/3}{\frac{5}{3}} = \frac{2}{5}$

$\frac{3}{3}$



$1 + \frac{2}{5} = \frac{7}{5}$

$\tau = R_{eq} C = \left(\frac{7}{5}\right)(1)$

$t_s = 5\tau = 7$

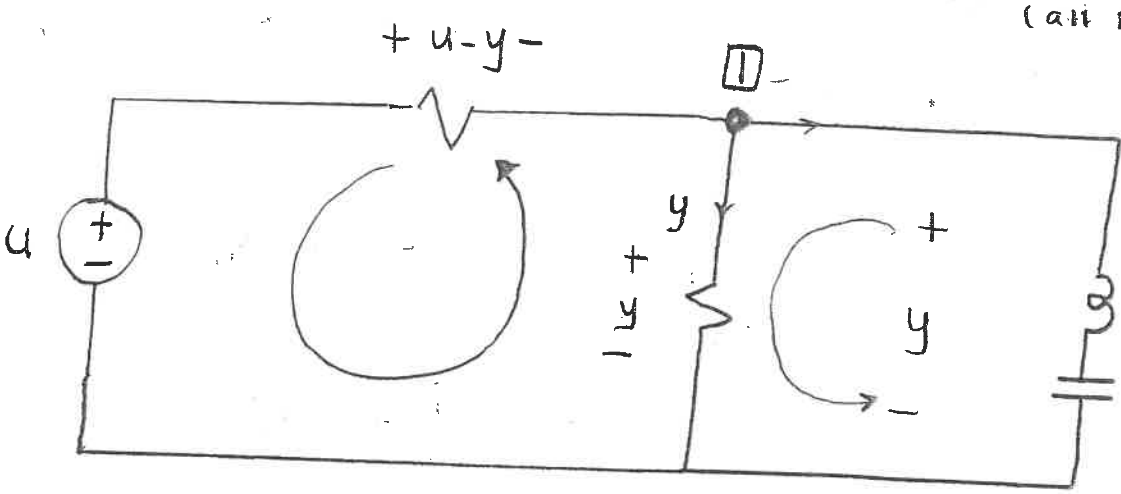
$\Longrightarrow$ $\boxed{\begin{array}{l} \tau = 7/5 \\ t_s = 7 \end{array}}$

Note: $\text{pole} = -\frac{1}{\tau} = -\frac{5}{7}$

$\Longrightarrow$ stable system

Problem # 3 Determine H, diffeq, τ, t_s, y_{ss} $\quad u = -5 + 10\sin(t + \ldots$

(all R = L = C = 1)



1 KCL: $\boxed{\left(\dfrac{u-y}{1}\right) = y + \dfrac{y}{s + \frac{1}{s}}}$ $\quad s + \frac{1}{s} = \dfrac{s^2+1}{s}$

$\Rightarrow u - y = y + \dfrac{sy}{s^2+1}$

$\Rightarrow u = y\left[1 + 1 + \dfrac{s}{s^2+1}\right]$ $\quad (1+1 = 2)$

$\Rightarrow (s^2+1)u = y\left[2s^2 + 2 + s\right]$

$\Rightarrow \boxed{H = \dfrac{y}{u} = \dfrac{\frac{1}{2}(s^2+1)}{s^2 + \frac{1}{2}s + 1}}$

$\boxed{\ddot{y} + \frac{1}{2}\dot{y} + y = \frac{1}{2}(\ddot{u} + u)}$

char eq:

$\Phi = s^2 + \frac{1}{2}s + 1 = 0 \Rightarrow s_{1,2} = \dfrac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4(1)(1)}}{2}$

$= -\frac{1}{4} \pm \sqrt{\frac{1}{16} - 1}$ $\quad \frac{16}{16}$

$= -\frac{1}{4} \pm j\dfrac{\sqrt{15}}{4} \Longrightarrow \tau = \dfrac{1}{|\text{Re pole}|} = \dfrac{1}{|-\frac{1}{4}|} = 4$

stable system

$t_s = 5\tau = 5(4) = 20$

$\boxed{\tau = 4, \ t_s = 20}$

MOTF

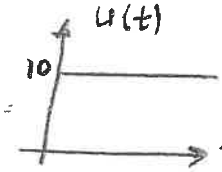
$\boxed{y_{ss} = -5\,H(0) + 10\,|H(j1)|\,\sin(t + 30° + \angle H(j1))}$

$H(0) = \frac{1}{2}$ $\qquad H(j1) = \dfrac{\frac{1}{2}((j1)^2+1)}{\text{den}} = \dfrac{\frac{1}{2}(-1+1)}{\text{den}} = 0$

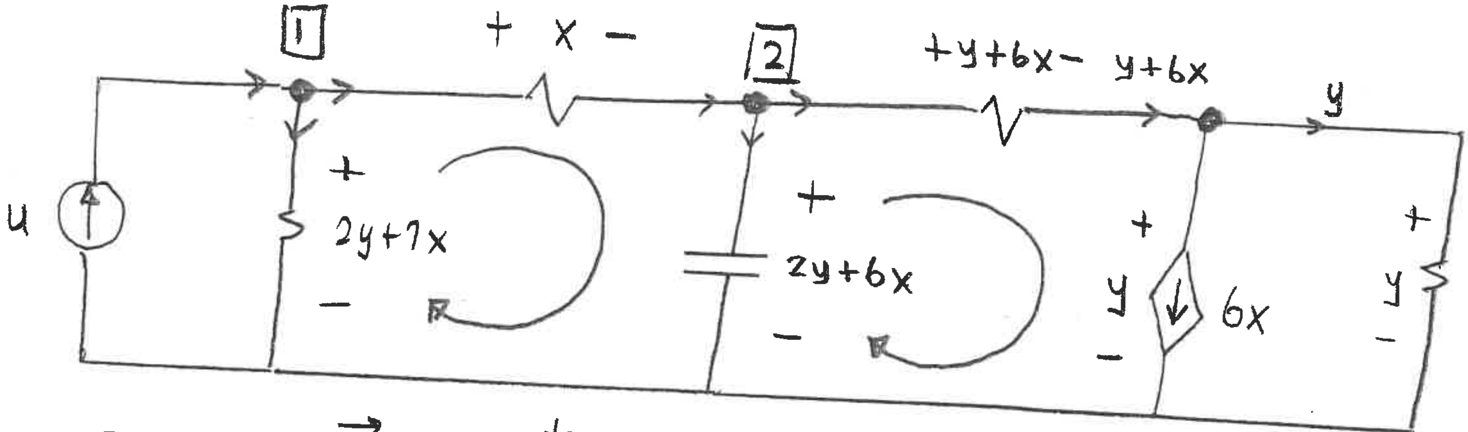
since H has zeros @ $\pm j1$!

Problem # 4

Determine H, diffeq, y



(all R=L=C=1)



1 KCL = $\boxed{u = \left(\frac{2y+7x}{1}\right) + \left(\frac{x}{1}\right)} \Rightarrow \boxed{u = 2y + 8x}$ ⊛

2 KCL = $\boxed{\left(\frac{x}{1}\right) = s(2y+6x) + (y+6x)} \Rightarrow y[2s+1] = x[1-6s-6]$

$= -x[6s+5]$

$\boxed{x = -\left(\frac{2s+1}{6s+5}\right) y}$ ⊛

⊛ $\Rightarrow \quad u = 2y + 8\left[-\left[\frac{2s+1}{6s+5}\right] y\right]$

$\Rightarrow (6s+5)u = [12s+10 - 16s - 8]y = [-4s+2]y$

$\Rightarrow \quad H = \frac{y}{u} = \frac{6(s+\frac{5}{6})}{-4(s-\frac{1}{2})} \Longrightarrow \boxed{H = \frac{y}{u} = -\frac{3}{2}\left[\frac{s+\frac{5}{6}}{s-\frac{1}{2}}\right]}$

pole @ $s = \frac{1}{2} \Rightarrow$ unstable system (pole in RHP!)

$\boxed{\dot{y} - \frac{1}{2}y = -\frac{3}{2}\left(\dot{u} + \frac{5}{6}u\right)}$

$Y = HU = -\frac{3}{2}\left[\frac{s+\frac{5}{6}}{s-\frac{1}{2}}\right]\underbrace{\left[\frac{10}{s}\right]}_{U} = \frac{A}{s} + \frac{B}{s-\frac{1}{2}} \Rightarrow \boxed{y = A + Be^{\frac{1}{2}t}}$ growing exponential

$A = \lim_{s\to 0} sY = sH(s)\frac{10}{s}\Big|_{s=0} = 10H(0) = 10\left(-\frac{3}{2}\left(\frac{5/6}{-1/2}\right)\right) = 10\left(\frac{5}{2}\right)$ $\boxed{A = 25}$

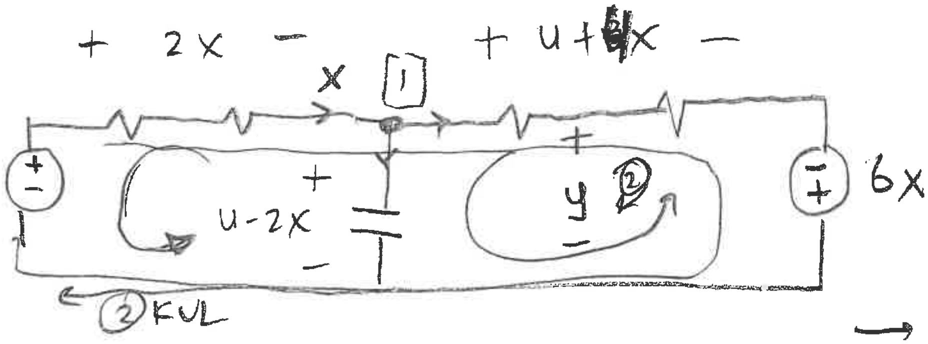
$B = \lim_{s\to\frac{1}{2}} (s-\tfrac{1}{2})Y = \left(-\frac{3}{2}\right)\left(s+\frac{5}{6}\right)\frac{10}{s}\Big|_{s=\frac{1}{2}} = \left(-\frac{3}{2}\right)\left(\frac{1}{2}+\frac{5}{6}\right)\frac{10}{(\frac{1}{2})}$ 8/6

$\boxed{B = -40}$

5/9

(Problem #4)

Alternate: via source transf



① KCL: $x = s(u-2x) + \frac{(u+4x)}{2}$

$$x[1 + 2s - 2] = u\left[s+\tfrac{1}{2}\right]$$

$$x = \left[\frac{s+\frac{1}{2}}{2s-1}\right]u$$

② KVL: $y = \left(\frac{u+4x}{2}\right)(1) - 6x$

$$= \frac{u}{2} + 2x - 6x$$

$$= \frac{u}{2} - 4x$$

$$= \frac{u}{2} - 4\left[\frac{s+\frac{1}{2}}{2s-1}\right]u$$

$$= \left[\frac{(2s-1) - 8(s+\frac{1}{2})}{2(2s-1)}\right]u$$

$$= \left[\frac{2s-1-8s-4}{4(s-\frac{1}{2})}\right]u$$

$$= \left[\frac{-6s-5}{4(s-\frac{1}{2})}\right]u \qquad H$$

$$= -\frac{3}{2}\left[\frac{s+5/6}{s-\frac{1}{2}}\right]u$$

✓✓ — same!

Sorry — I know this is crowded! AAR

$\boxed{\text{Problem \#5}}$

Determine y_{1ss}, y_1, t_{s1}, y_{2ss}, y_2, t_{s2}

$$H_1(s) = \frac{s}{(s+2)(s^2-6s+25)}$$

$\tau_1 = \frac{1}{2}$

$t_{s1} = 5\tau_1 = \frac{5}{2}$

$\frac{6 \pm \sqrt{36-4(1)(25)}}{2} = 3 \pm j4$ ← H_1 is unstable (has poles in RHP)

$u_1(t) = -10 + 50\cos(0.01t - 45^\circ) + 8\sin(5t+30^\circ)$

$U_1 = \frac{-10}{s} + \frac{}{s^2+10^{-4}} + \frac{}{s^2+25}$

MOTF

$y_{1ss} = -10\,H_1(0) + 50\,|H_1(j0.01)|\cos(0.01t - 45^\circ + \angle H_1(j0.01)) + 8\,|H_1(j5)|\sin(5t+30^\circ+\angle H_1(j5))$ + unstable term due to poles @ $+3\pm j4$ (see PFE below)

$H_1(0) = 0$ since H_1 has a zero @ $s=0$!

$H(j0.01) \cong \frac{s}{(2)(25)}\Big|_{s=j0.01=0.01e^{j90^\circ}} = \frac{1}{50}(0.01e^{j90^\circ}) = \frac{1}{5000}e^{j90^\circ}$

$H_1(j5) = \frac{(j5)}{(j5+2)(\cancel{-25} - j6(5) + \cancel{25})} = \frac{5e^{j90^\circ}}{(\sqrt{25+4}\,e^{j\tan^{-1}(\frac{5}{2})})(30e^{-j90^\circ})}$

$-j30 = 30e^{-j90^\circ}$

$= \left(\frac{5}{\sqrt{29}}\right)\left(\frac{1}{30}\right)e^{j[90^\circ - \tan^{-1}(\frac{5}{2}) - (-90^\circ)]}$

$|H_1(j5)|$ $\qquad \angle H_1(j5)$

$C = \lim_{s\to 3-j4}(s-3+j4)Y_1(s)$

$$Y_1 = H_1U_1 = \left[\frac{\cancel{s}}{(s+2)(s^2-6s+25)}\right]\frac{}{\cancel{s}(s^2+10^{-4})(s^2+25)}$$

$$= \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s-3+j4} + * + \frac{D}{s-j0.01} + * + \frac{E}{s-j5} + *$$

$y_1 = A + Be^{-2t} + 2|C|e^{3t}\cos(4t+\angle C) + 2|D|\cos(0.01t+\angle D) + 2|E|\cos(5t+\angle E)$

$D = \lim_{s\to j0.01}(s-j0.01)Y_1$

$E = \lim_{s\to j5}(s-j5)\,Y_1(s)$

$A = \lim_{s\to 0} sY_1 = -10\,H_1(0)$

$$H_2(s) = \frac{s^2+1}{(s+1)(s^2+10s+100)}$$

$\tau_2 = 1$

$t_{s2} = 5\tau_2 = 5(1) = 5$

$-5 \pm j5\sqrt{3}$

$u_2(t) = 7 - \cos(t+20^\circ) + 6\sin(100t+135^\circ)$

$U_2 = \frac{7}{s} - \frac{}{s^2+1} + \frac{}{s^2+10^4} = \frac{}{s(s^2+1)(s^2+10^4)}$

$y_{2ss} = 7H_2(0) - |H_2(j1)|\cos(t+20^\circ+\angle H_2(j1)) + 6\,|H_2(j100)|\sin(100t+135^\circ+\angle H_2(j100))$

cos (" + " + " -90°)

$H_2(0) = \frac{1}{(1)(100)}$

$H_2(j1) = 0 = 0e^{j0}$ since H_2 has zero @ $\pm j1$

$H_2(j100) \cong \frac{1}{s}\Big|_{s=j100=100e^{j90^\circ}} = \frac{1}{100e^{j90^\circ}} = 0.01e^{-j90^\circ}$

$$Y_2 = H_2U_2 = \left[\frac{\cancel{s^2+1}}{(s+1)(s^2+10s+100)}\right]\left[\frac{}{s\cancel{(s^2+1)}(s^2+10^4)}\right] = \frac{A}{s} + \frac{B}{s+1} + \left[\frac{C}{s+5-j5\sqrt{3}} + *\right]$$

$$+ \left[\frac{D}{s-j100} + *\right]$$

$y_2 = A + Be^{-t} + 2|C|e^{-5t}\cos(5\sqrt{3}t + \angle C) + 2|D|\cos(100t + \angle D)$

y_{2ss}

$A = \lim_{s\to 0} sY_2 = 7H_2(0)$

$D = \lim_{s\to j100}(s-j100)Y_2$ ← related to $H_2(j100)$!

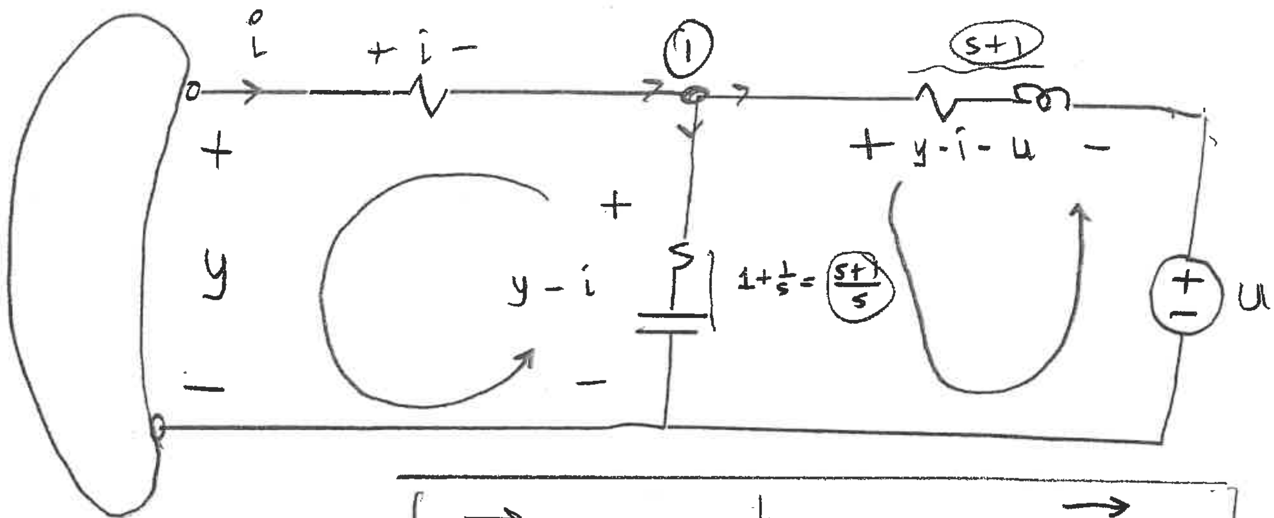
$= \frac{6|H_2|}{2}e^{j[135 + \angle H_2 - 90^\circ]}$

$\boxed{6/9}$

Problem #6

Determine an s-domain Thevenin Equivalent at y

(all R = L = C = 1)



$$\textcircled{1}\ \text{KCL} = \boxed{\ i = \frac{s(y-i)}{s+1} + \left(\frac{y-i-u}{s+1}\right)\ }$$

$$y\left[\frac{s}{s+1} + \frac{1}{s+1}\right] = i\left[1 + \frac{s}{s+1} + \frac{1}{s+1}\right] + u\left[\frac{1}{s+1}\right]$$

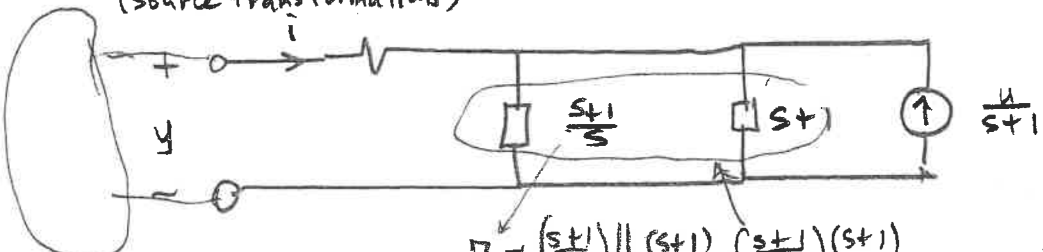
$$y\,[s+1] = i\,[\underbrace{s+1+s+1}_{2s+2}] + u\,[1]$$

$$\boxed{y = [2]\,i + \left[\frac{1}{s+1}\right]u}$$

$\boxed{Z_{th} = 2}$ V_{th}

WOW!

Alternate Method:
(source transformations)



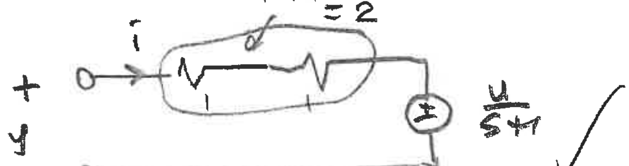
admittances in parallel add

$$Y = \frac{1}{Z} = Y_1 + Y_2$$
$$= \frac{s}{s+1} + \frac{1}{s+1} = \frac{s+1}{s+1} = 1$$
$$\Rightarrow Z = 1$$

$$Z = \left(\frac{s+1}{s}\right) \| (s+1) = \frac{\left(\frac{s+1}{s}\right)(s+1)}{\frac{s+1}{s} + s+1} = \frac{(s+1)(s+1)}{s+1+s^2+s} = \frac{s^2+2s+1}{s^2+2s+1} = 1$$

WOW!

$R_{th} = 1 + 1$
$= 2$



✓

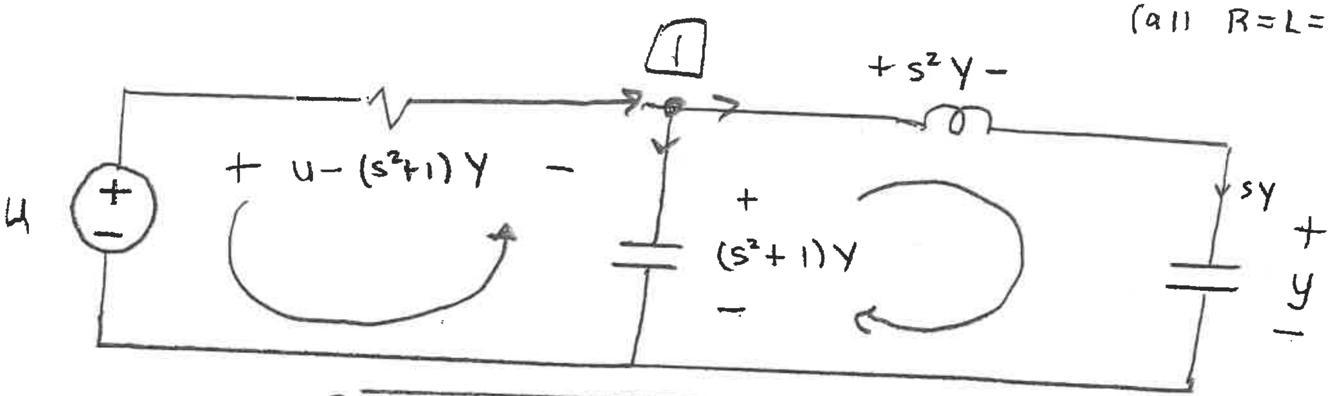
7/9

Problem # 7

Determine H & y_{ss}

$u = -5 + 2\sin(t + 30°) - 3\cos(100t - 135°)$

(all $R = L = C = 1$)



$$\textcircled{1} \text{ KCL} = \boxed{\left(\frac{u - (s^2+1)Y}{1}\right) = s\left[(s^2+1)Y\right] + (sY)}$$

$$y\left[s^2 + 1 + s^3 + s + s\right] = u$$

$$\boxed{H = \frac{y}{u} = \frac{1}{s^3 + s^2 + 2s + 1}}$$

$$y_{ss} = -5\underbrace{H(0)}_{1} + 2\,\overbrace{|H(j1)|}^{1}\sin(t + 30° + \overbrace{\angle H(j1)}^{-90°}) - 3\overbrace{|H(j100)|}^{10^{-6}}\cos(100t - 135° + \overbrace{\angle H(j100)}^{-270°})$$

$H(0) = 1$

$$H(j1) = \frac{1}{(j)^3 + (j)^2 + 2j + 1} = \frac{1}{-j \cancel{-1} + j2 \cancel{+1}}$$

$$= \frac{1}{\underbrace{j}_{1e^{j90°}}}$$

$$= 1e^{-j90}$$

$$H(j100) \cong \left.\frac{1}{s^3}\right|_{s = j100 = 10^2 e^{j90°}} = \frac{1}{10^6 e^{j270}}$$

$$= 10^{-6} e^{-j270}$$

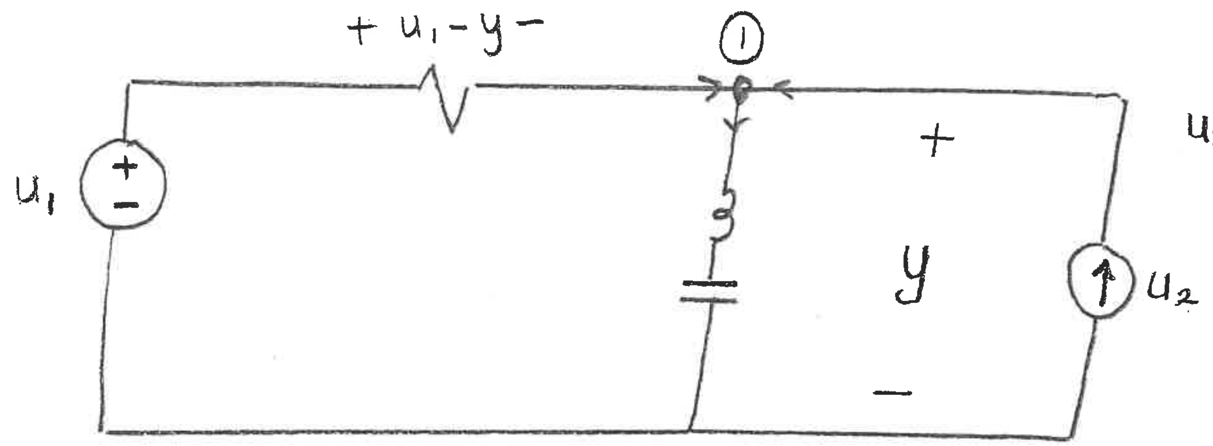
Problem # 8

Determine H_1, H_2, ts_1, y_{1ss}

$u_1 = 10 - 8\sin(t+30^\circ) + 7\cos(100t+4$

$u_2 = 0$

(all $R=L=C=1$



① KCL = $\left(\frac{u_1 - y}{1}\right) + \left(u_2\right) = \frac{y}{s+\frac{1}{s}}$ $\quad \frac{s^2+1}{s}$ $\quad = \frac{s\,y}{s^2+1}$

$$u_1 + u_2 = y\left[1 + \frac{s}{s^2+1}\right]$$

$$y\left[s^2+1+s\right] = (s^2+1)(u_1+u_2)$$

$$y = \left[\frac{s^2+1}{s^2+s+1}\right](u_1+u_2)$$

$H_1 = H_2$

poles $= -\frac{1}{2} \pm j\frac{\sqrt{3}}{2} \Longrightarrow \tau_1 = \frac{1}{|Re\ Poles|} = \frac{1}{|-\frac{1}{2}|} = 2$

$ts_1 = 5\tau_1 = 5(2) = 10$

$y_{1ss} = 10\,H_1(0) - 8\,|H_1(j1)|\sin(t+30^\circ + \boxed{\angle H_1(j1)}) + 7\,|H_1(j100)|\cos(100t + 45^\circ + \angle H_1(j100)$

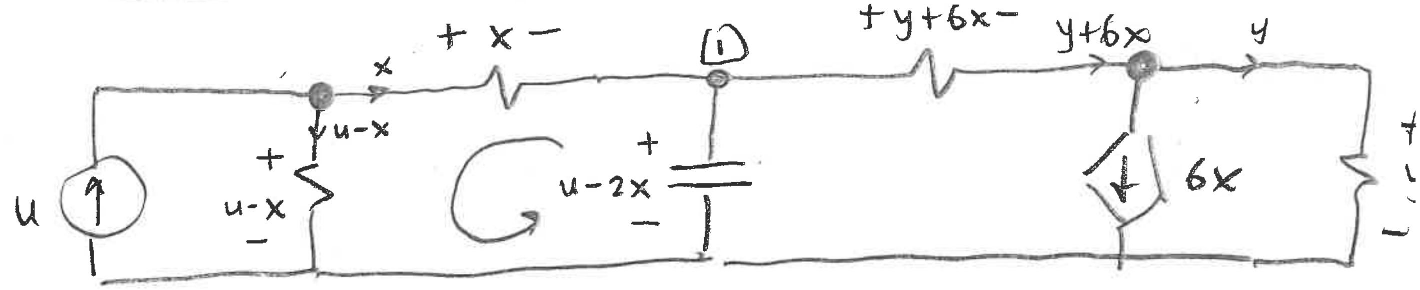
$H_1(0) = 1$

$H_1(j1) = 0$ since H_1 has zeros @ $\pm j1$

$H_1(j100) \cong 1 = 1 e^{j0^\circ}$

9/9

Problem #4 (Alternate Solution)



(1) KCL: $x = s(u-2x) + (y+6x) \Rightarrow x(1+2s-6) = su + y$

$$\Rightarrow \boxed{x = \frac{y+su}{2s-5}}$$

AAR SOLUTIONS'

EXAM #2

Fall 2023

Fri 12-8-23

[redacted]

Final Exam Rules

Final Exam
Fall 2023
Fri 12-15-23

1. One $8\frac{1}{2}$" x 11" sheet permitted; otherwise closed books, closed notes, closed phone. open minds!

2. Calculator permitted (no computer!!; no phone!!!)

3. NO PHONES! Phones should not be visible at all! A visible phone will result in a zero grad for the exam!

4. Write full name legibly on each page provided.

5. Please show all work. Write your solutions on the pages provided. (No other pages/paper should be used!)

6. Clearly label voltage & currents on circuits provided.

7. Use variables provided! (No additional variables!)

8. Unreadable work will receive NO CREDIT.

9. Please place important equations & answers within box

10. Please be careful with your algebra, signs, etc.

11. Please turn in solutions to me at end of period!

12. PLEASE DO NOT CHEAT!!!

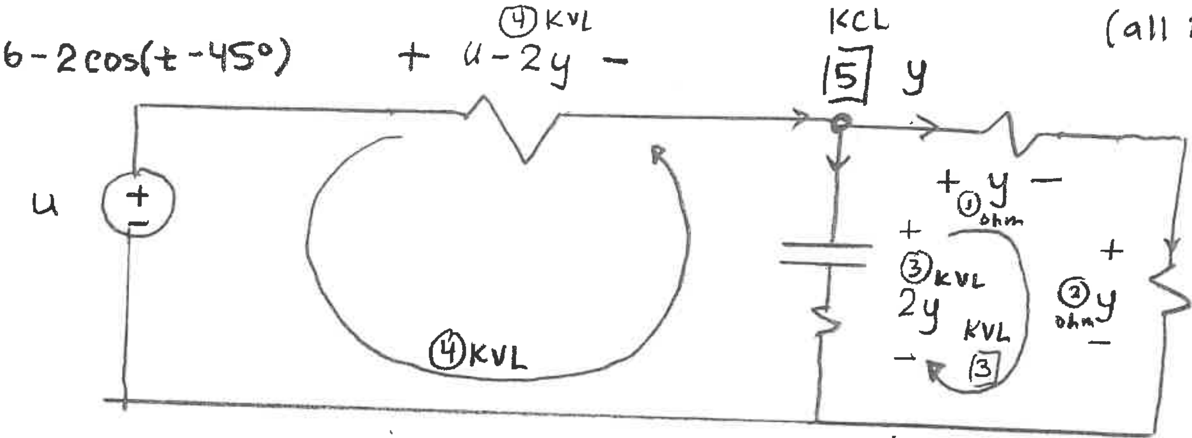
Dr. A.A. Rodriguez
EE202

AAR SOLUTIONS!

Problem #1

Determine H, diff eq, t_s, y_{ss}, y

$u = -6 - 2\cos(t - 45°)$ (all $R = L = C = 1$)



5 KCL: $\boxed{\left(\frac{u-2y}{1}\right) = (y) + \left(\frac{2y}{\frac{1}{s}+1}\right)} = y + \frac{2sy}{s+1}$ $\quad \left(\frac{s+1}{s}\right)$

$u = y\left[\cancel{2+1}^{\,3} + \frac{2s}{s+1}\right]$

$\Rightarrow (s+1)u = y[3s+3+2s] = y[5s+3]$

$\Rightarrow \boxed{H = \frac{y}{u} = \frac{\frac{1}{5}(s+1)}{(s+\frac{3}{5})}} \Rightarrow \dot{y} + \frac{3}{5}y = \frac{1}{5}(\dot{u} + u)$

$\Rightarrow$ pole $= -\frac{3}{5}$ $\quad -\frac{3}{5}$ ← stable $\Rightarrow \tau = \frac{1}{|\text{Re pole}|} = \frac{1}{|-\frac{3}{5}|} = \frac{5}{3}$

$\boxed{t_s = 5\tau = \frac{25}{3}}$

HOTF

$\boxed{y_{ss} = -6H(0) - 2|H(j1)|\cos(t - 45° + \angle H(j1))}$

$H(0) = \frac{1}{3}$ $\qquad H(j1) = \frac{\frac{1}{5}(j1+1)}{(j1+\frac{3}{5})}$ $\quad 1+j1 = \sqrt{2}e^{j45°}$ $\quad \frac{3}{5}+j1 = \sqrt{\frac{9}{25}+1}\; e^{j\tan^{-1}\left(\frac{1}{\frac{3}{5}}\right)}$

$\sqrt{2}$, $45°$, 1, 1, $\frac{3}{5}$

$U = -\frac{6}{s} - \frac{\quad}{s^2+1} = \frac{\quad}{s(s^2+1)}$ $\quad s = \pm j1$

$|H(j1)| = \frac{|\text{top}|}{|\text{bottom}|} = \frac{\frac{1}{5}\sqrt{2}}{\sqrt{\frac{9}{25}+1}}$ $\qquad \angle H(j1) = \angle\text{top} - \angle\text{bottom} = 45° - \tan^{-1}\left(\frac{1}{\frac{3}{5}}\right)$

$Y = HU = \left[\frac{\quad}{s+\frac{3}{5}}\right]\left[\frac{\quad}{s(s^2+1)}\right] = \frac{A}{s} + \frac{B}{s+\frac{3}{5}} + \left[\frac{C}{s-j1} + *\right] \Rightarrow y = A + Be^{-\frac{3}{5}t} + 2|C|\cos(t + \angle C)$

[stable term]

$A = \lim_{s\to 0} sY = -6H(0)$ $\qquad C = \lim_{s\to j1}(s-j1)Y$ $\qquad -6H(0)$ $\qquad y_{ss}$

$-2|H(j1)|\cos(t - 45° + \angle H(j1))$

$= 2|H(j1)|\cos(t - 45° + \angle H(j1) \pm 180°)$

Note: $B = \lim_{s\to -\frac{3}{5}}(s+\frac{3}{5})Y$

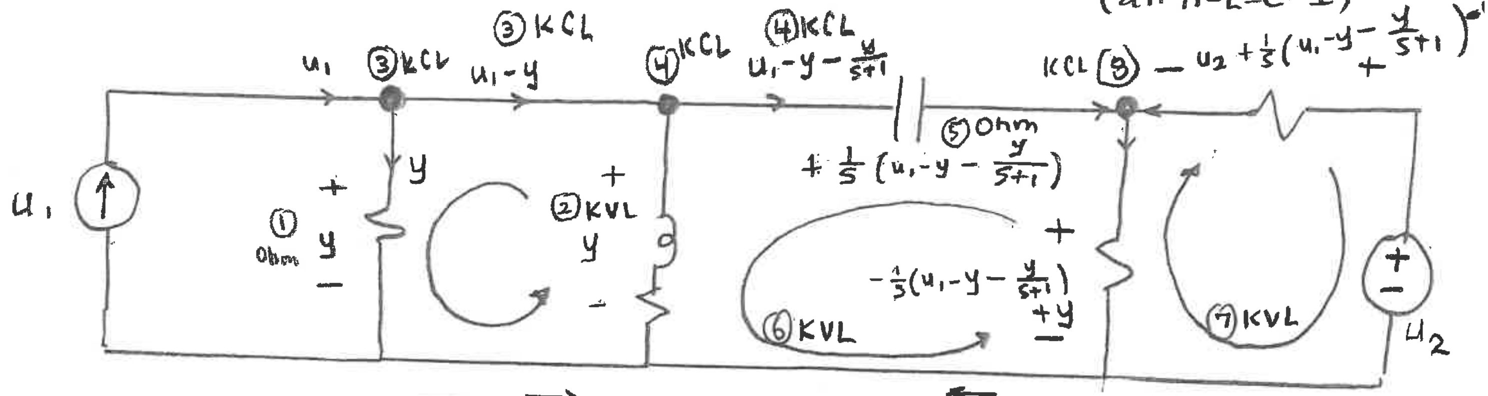
$\Rightarrow C = |H(j1)|e^{j(-45° + \angle H(j1) \pm 180°)}$

Final Fall 2023

Problem # 2

Determine $H_1(0)$, $H_2(0)$, $H_1(\infty)$, $H_2(\infty)$, poles, t_s

(all $R=L=C=1$)



⑧ KCL:

$$\left(u_1 - y - \frac{y}{s+1}\right) + \left(\frac{u_2 + \frac{1}{s}\left(u_1 - y - \frac{y}{s+1}\right) - y}{1}\right) = \left(\frac{-\frac{1}{s}\left(u_1 - y - \frac{y}{s+1}\right) + y}{1}\right)$$

$$u_1\left[1 + \frac{1}{s} + \frac{1}{s}\right] + u_2[1] = y\left[1 + \frac{1}{s+1} + \frac{1}{s} + \frac{1}{s(s+1)} + 1 + \frac{1}{s} + \frac{1}{s(s+1)} + \right.$$

$$u_1\left[1 + \frac{2}{s}\right] + u_2[1] = y\left[3 + \frac{1}{s+1} + \frac{2}{s} + \frac{2}{s(s+1)}\right]$$

$\left(\frac{s+2}{s}\right)$

$\times (s)(s+1)$

$$u_1[(s+2)(s+1)] + u_2[s(s+1)] = y[3s(s+1) + s + 2(s+1) + 2]$$

$6s \qquad 4$

$$u_1[s^2 + 3s + 2] + u_2 s(s+1) = y[3s^2 + 3s + s + 2s + 2 + 2]$$

$$= 3y\left[s^2 + 2s + \frac{4}{3}\right]$$

$$\Rightarrow \boxed{y = \left[\frac{\frac{1}{3}(s^2+3s+2)}{s^2 + 2s + \frac{4}{3}}\right] u_1 + \left[\frac{\frac{1}{3}s(s+1)}{s^2 + 2s + \frac{4}{3}}\right] u_2}$$

$H_1 \qquad H_2$

$\boxed{H_1(0) = \frac{1}{2} \qquad H_2(0) = 0}$ $\boxed{H_1(\infty) = H_2(\infty) = \frac{1}{3}}$

poles $=$ $\Phi(s) = s^2 + 2s + \frac{4}{3} = 0 \Rightarrow s_{1,2} = \frac{-2 \pm \sqrt{4 - 4(1)(\frac{4}{3})}}{2(1)} = -1 \pm \sqrt{1 - \frac{4}{3}}$ $-\frac{1}{3}$

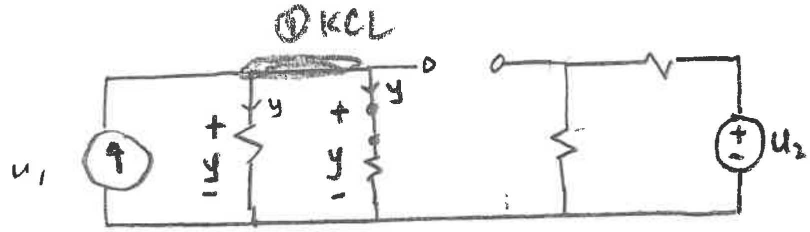
$$= -1 \pm j\frac{1}{\sqrt{3}}$$

$$\tau = \frac{1}{|\text{Re poles}|} = \frac{1}{|-1|} = 1$$

$$t_s = 5\tau = 5(1) = 5$$

NOTE: Above was the long (hard) way! Lets see how to get answers much faster --- without KVL, KCL, Ohm!

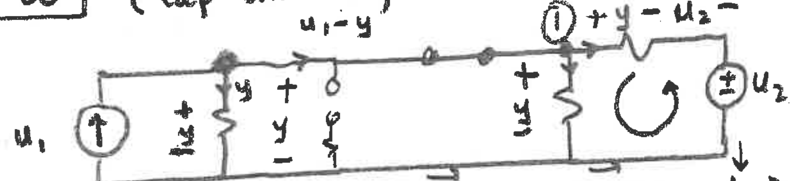
$\boxed{s=0}$ (Cap open, Inductor shorted)



① KCL: $u_1 = y + y = 2y \implies y = \frac{1}{2}u_1 \implies \boxed{H_1(0) = \frac{1}{2} \quad H_2(0) = 0}$

Agrees with what we got on reverse side!

$\boxed{s=\infty}$ (Cap shorted, Inductor open)

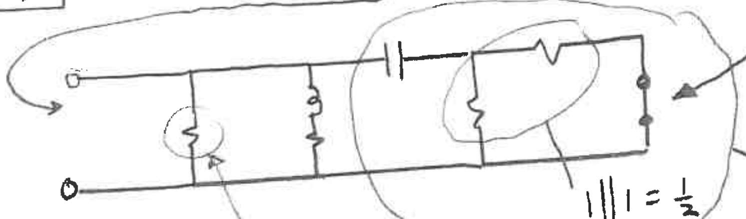


① KCL: $u_1 - y = \frac{y - u_2}{1} + y \implies u_1 + u_2 = 3y \implies y = \frac{1}{3}u_1 + \frac{1}{3}u_2$

$$\boxed{H_1(\infty) = H_2(\infty) = \frac{1}{3}}$$

Agrees with what we got on reverse side!

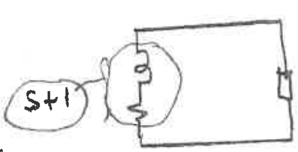
$\boxed{t_s = ?}$ (Set $u_1 = 0$ (open); Set $u_2 = 0$ (short))



$1 \| 1 = \frac{1}{2}$

$\frac{1}{2} + \frac{1}{s} = \frac{\frac{1}{2}s + 1}{s}$

These are in parallel



$$\| \frac{\frac{1}{2}s+1}{s} = \frac{\frac{\frac{1}{2}s+1}{s}}{1 + \frac{\frac{1}{2}s+1}{s}} = \frac{\frac{1}{2}s+1}{s + \frac{1}{2}s + 1} = \frac{\frac{1}{2}s+1}{\frac{3}{2}s+1}$$

$$Z_{total} = s + 1 + \frac{\frac{1}{2}s+1}{\frac{3}{2}s+1} = \frac{(s+1)(\frac{3}{2}s+1) + \frac{1}{2}s + 1}{\frac{3}{2}s+1}$$

$$\frac{3}{2}s^2 + \frac{5}{2}s + 1 + \frac{1}{2}s + 1 = \frac{3}{2}s^2 + \frac{6}{2}s + 2$$

$$= \frac{3}{2}\left[s^2 + 2s + \frac{4}{3}\right]$$

$$\Phi(s) = s^2 + 2s + \frac{4}{3} = 0 \implies s_{1,2} = -1 \pm j\frac{1}{\sqrt{3}} \implies \tau = \frac{1}{|\text{Re pole}|} = 1$$

what we got on reverse side!

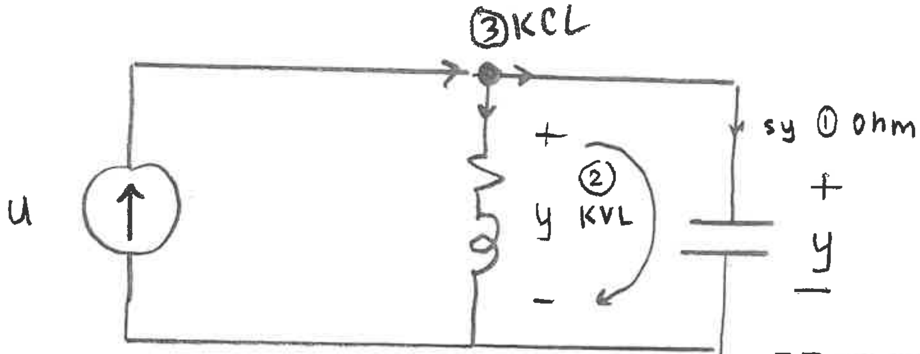
$$t_s = 5\tau = 5(1) = 5$$

NOTE: ALL THIS IS MUCH EASIER THAN WHAT WE DID ON REVERSE SIDE!

Problem # 3

Determine H, diff eq, ts, yss

(all R = L = C = 1)



$u = 4 - 3\sin(t + 135°) \xRightarrow[\text{stability}]{\text{given}}$ $\boxed{y_{ss} = 4H(0) - 3|H(j1)|\sin(t + 135° + \angle H(j1))}$ MOTF

↑ we know system (circuit) is stable since it consists only of independent sources & passive elements (i.e. R, L, & Cs).

--- we find H from cct analysis!

③ KCL =

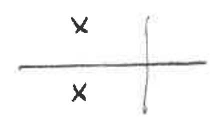
$$\boxed{u = \frac{y}{1+s} + sy} = y\left[\frac{1}{s+1} + s\right]$$

transf function

$$\Rightarrow (s+1)u = y[1 + s^2 + s] \Rightarrow \boxed{H = \frac{y}{u} = \frac{s+1}{s^2+s+1}}$$

diff eq = $\ddot{y} + \dot{y} + y = \dot{u} + u$

poles = $s_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$



all poles in LHP ⟺ system is stable

$H(0) = 1$

$$H(j1) = \frac{j1 + 1}{-1 + j1 + 1} = \frac{(1 + j1)}{(j1)}$$

$\sqrt{2}$, 45°, 1, 1

$1\angle 90°$

$$\Rightarrow \tau = \frac{1}{|\text{Re poles}|} = \frac{1}{\frac{1}{2}} = 2$$

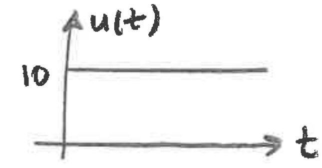
$$|H(j1)| = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$t_s = 5\tau = 5(2) = 10$$

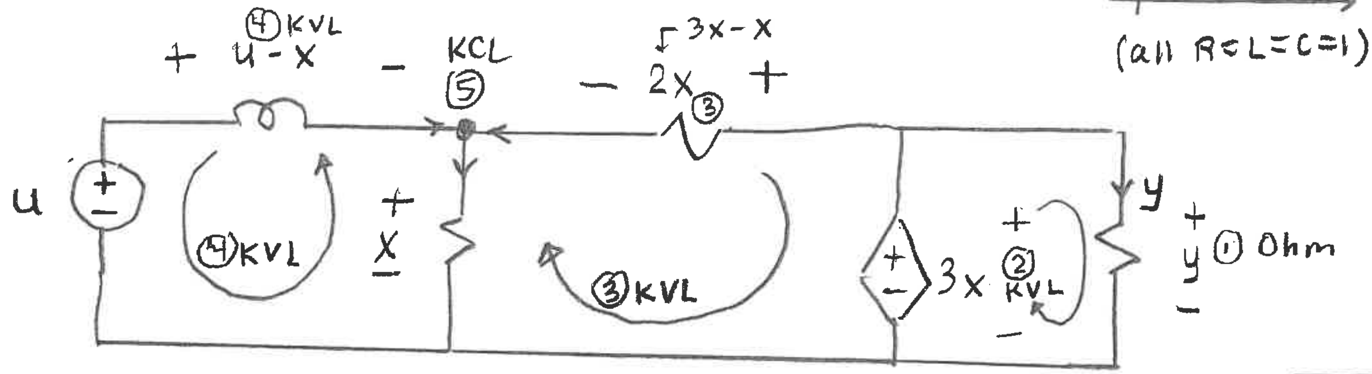
$$\angle H(j1) = 45° - 90$$

Problem # 4

Determine H, diffeq, y



(all $R=L=C=1$)



2 KVL: $3x = y \Rightarrow \boxed{x = \frac{y}{3}}$

WOW! That was easy!

5 KCL $= \boxed{\left(\frac{u-x}{s}\right) + \left(\frac{2x}{1}\right) = \left(\frac{x}{1}\right)}$

$$\Rightarrow \frac{u}{s} = x\left[\frac{1}{s} - 2 + 1\right] = x\left[\frac{1}{s} - 1\right]$$

$$= x\left[\frac{1-s}{s}\right]$$

$$= \frac{y}{3}\,\frac{[1-s]}{s}$$

$$\Rightarrow \boxed{H = \frac{y}{u} = \frac{-3}{s-1}} \Rightarrow$$

diff eq: $\dot{y} - y = -3u$

pole: $s = 1$

pole in RHP $\Rightarrow$ system is unstable

$$Y = HU = \left[\frac{-3}{s-1}\right]\left[\frac{10}{s}\right] = \frac{A}{s} + \frac{B}{s-1}$$

$U = \mathcal{L}\{10\}$

$$\Rightarrow \boxed{y(t) = A + Be^{t}}$$

expected from MOTF

$$A = \lim_{s\to 0} sY = \cancel{s} H \frac{10}{\cancel{s}}\Big|_{s=0} = 10\,H(0) = 10\left(\frac{-3}{-1}\right)$$

$$= (10)(3)$$

$$= 30$$

$$B = \lim_{s\to 1}(s-1)Y = \cancel{(s-1)}\left(\frac{-3}{\cancel{s-1}}\right)\frac{10}{s}\Big|_{s=1}$$

$$= (-3)\left(\frac{10}{1}\right)$$

$$= -30$$

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Problem # 5

Determine y_{ss}, y, t_s

$$H(s) = \frac{s(s^2+4)}{(s+2)(s^2+2s+10)(s^2-s+1)}$$

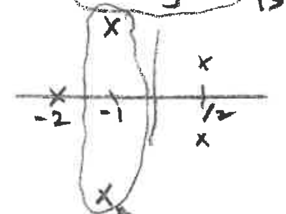
pole = −2 (stable) poles = $\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ (unstable)

$$u(t) = 5 - 6\sin(2t+45^\circ) + 7\cos(100t+270^\circ)$$

(compute all important coefficients as discussed in class)

Poles: $s_{1,2} = \frac{-2 \pm \sqrt{4-4(1)(10)}}{2(1)} = -1 \pm \sqrt{1-10}$
$= -1 \pm j3$ (stable)

$$U = \frac{5}{s} - \frac{\quad}{s^2+4} + \frac{\quad}{s^2+10^4} = \frac{\quad}{s(s^2+4)(s^2+10^4)}$$



$$Y = HU = \left[\frac{s(s^2+4)}{(s+2)(s+1\mp j\sqrt{3})(s-\frac{1}{2}\mp j\frac{\sqrt{3}}{2})}\right]\left[\frac{\quad}{s(s\mp j2)(s\mp j100)}\right]$$

$$= \frac{A}{s} + \frac{B}{s+2} + \left[\frac{C}{s+1-j\sqrt{3}} + *\right] + \left[\frac{D}{s-\frac{1}{2}-j\frac{\sqrt{3}}{2}} + *\right]$$
$$+ \left[\frac{E}{s-j2} + *\right]$$
$$+ \left[\frac{F}{s-j100} + *\right]$$

slow poles

$$\tau = \frac{1}{|\mathrm{Re\,poles}|} = \frac{1}{|-1|} = 1$$

$$t_s = 5\tau = 5(1) = 5\text{ sec}$$

Note: It will take 5sec for the output y of the unstable system H to reach y_{ss}

$$y(t) = A + \overset{\text{stable}}{Be^{-2t}} + \overset{\text{stable}}{2|C|e^{-t}\cos(\sqrt{3}t + \angle C)}$$
$$+ 2|D|e^{\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t + \angle D\right) \leftarrow \text{unstable}$$
$$+ 2|E|\cos(2t + \angle E)$$
$$+ 2|F|\cos(100t + \angle F)$$

from HOTF

$$y_{ss} = 5H(0) - 6|H(j2)|\sin(2t + 45^\circ + \angle H(j2)) + 7|H(j100)|\cos(100t + 270^\circ + \angle H(j100))$$

$6|H(j2)|\cos(2t + 45 + \angle H(j2) + 90^\circ)$ $= 2|E|\cos(2t + \angle E)$

+ unstable term $2|D|e^{\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t + \angle D\right)$ from PFE!

$$A = \lim_{s\to 0} sY \overset{\text{HOTF}}{=} 5H(0) = 5(0) = 0$$

$$D = \lim_{s\to \frac{1}{2}+j\frac{\sqrt{3}}{2}} \left(s - \frac{1}{2} - j\frac{\sqrt{3}}{2}\right)Y$$

$$E = \frac{6|H(j2)|}{2} e^{j(45^\circ + \angle H(j2) + 90^\circ)}$$

$$E = \lim_{s\to j2} (s-j2)Y$$

$$F = \lim_{s\to j100} (s-j100)Y = \frac{7|H(j100)|}{2} e^{j(270^\circ + \angle H(j100))}$$

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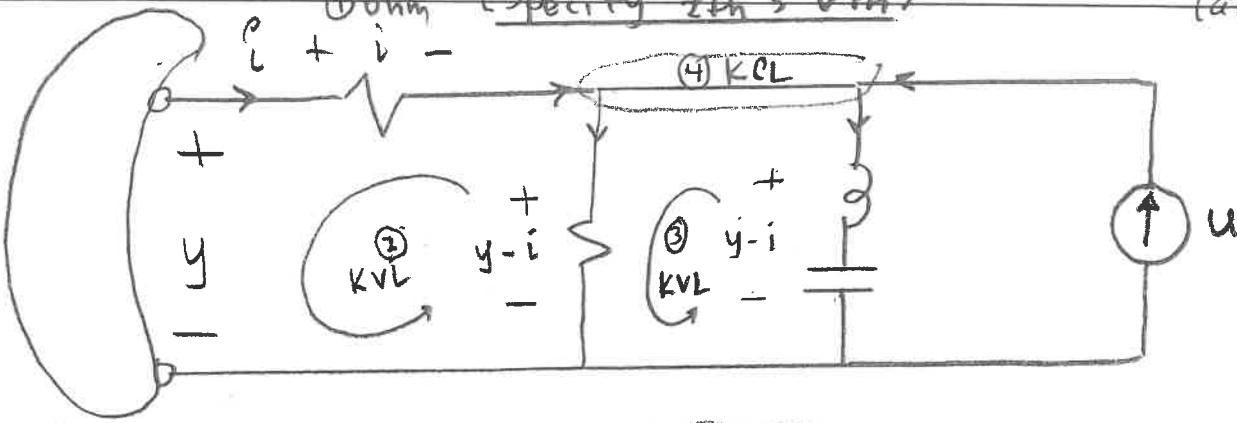
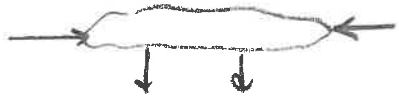
Problem # 6

Determine an s-domain Thevenin Equivalent at y (Specify Z_{th} & V_{th}) (all $R=L=C=1$)

① ohm



④ KCL =



$$\boxed{\vec{i} + \overleftarrow{u} = \left(\frac{y-i}{1}\right) + \left(\frac{y-i}{s+\frac{1}{s}}\right)}$$

$$i\left[\cancel{1}+1+\frac{1}{s+\frac{1}{s}}\right] + u = y\left[1+\frac{1}{s+\frac{1}{s}}\right]$$

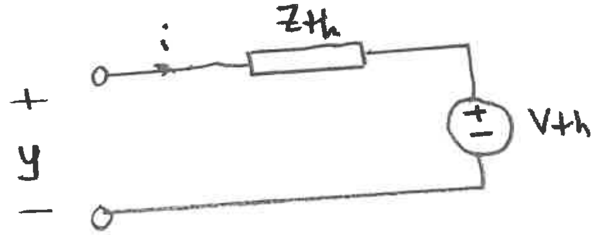
2 $\qquad \frac{s}{s^2+1} \qquad \frac{s}{s^2+1}$

$$i\left[2s^2+2+s\right] + u\left[s^2+1\right] = y\left[s^2+1+s\right]$$

$$2\left(s^2+\tfrac{1}{2}s+1\right)$$

$$\boxed{y = \left[\frac{2\left(s^2+\frac{1}{2}s+1\right)}{s^2+s+1}\right] i + \left[\frac{s^2+1}{s^2+s+1}\right] u}$$

Z_{th} $\qquad$ V_{th}

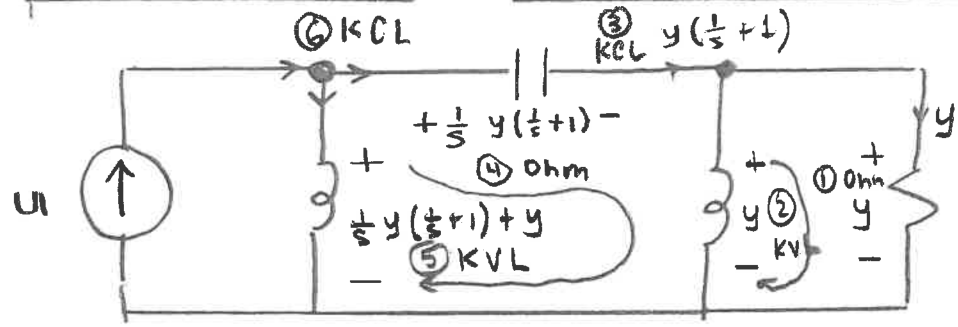


Problem # 7

Determine H & y_{ss}

[Compute all coefficients in y_{ss} (approximately)]

(all $R=L=C=1$)



$$u = -7 + 8\cos(0.01t - 180^\circ) - 9\sin(100t + 20^\circ)$$

$\downarrow$ MOTF

$$y_{ss} = -7H(0) + 8\,|H(j0.01)|\cos(0.01t - 180^\circ + \angle H(j0.01))$$
$$-9\,|H(j100)|\sin(100t + 20^\circ + \angle H(j100))$$

Note: H is found from circuit analysis!

⑥ KCL = $\overrightarrow{\downarrow}$

$$\boxed{u = \left(\frac{\frac{1}{s}y(\frac{1}{s}+1) + y}{s}\right) + \left(y(\frac{1}{s}+1)\right)}$$

$$= y\left[\frac{1}{s^3} + \frac{1}{s^2} + \frac{1}{s} + \frac{1}{s} + 1\right]$$

$\frac{2}{s}$

$$s^3 u = y\left[1 + s + 2s^2 + s^3\right]$$

$$\boxed{H = \frac{y}{u} = \left[\frac{s^3}{s^3 + 2s^2 + s + 1}\right]}$$

$H(0) = 0$ (so dc term of u gets "Killed" i.e. absorbed by H)

$$H(j0.01) \cong s^3\big|_{s=j0.01} = 10^{-6}e^{j270} \sim \angle H(j0.01)$$

$= 0.01e^{j90^\circ}$

$= 10^{-2}e^{j90^\circ}$

$\sim |H(j0.01)|$

$$H(j100) \cong H(\infty) = 1 = 1e^{j0^\circ} \sim \angle H(j100)$$

$\sim |H(j100)|$

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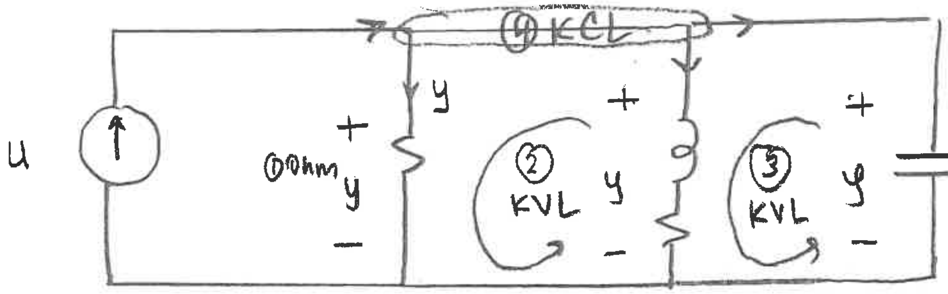
Problem #8

Determine H, ts, yss

$u = -8 + 9\sin(t - 45°)$

(all R=L=C=1)



$$\boxed{y_{ss} = -8H(0) + 9|H(j1)|\sin(t - 45° + \angle H(j1))}$$

MOTF

H is found from circuit analysis

④ KCL =

$$\boxed{\vec{u} = \left(\frac{y}{1}\right) + \left(\frac{y}{s+1}\right) + (sy)}$$

$$= y\left[1 + \frac{1}{s+1} + s\right]$$

$$(s+1)u = y\left[s+1 + 1 + s^2 + s\right]$$

$$s^2 + 2s + 2$$

poles $= -1 \pm j1$

$\tau = \frac{1}{|\text{Re poles}|} = \frac{1}{|-1|} =$

$t_s = 5\tau = 5$

$$\boxed{H = \frac{y}{u} = \left[\frac{s+1}{s^2 + 2s + 2}\right]}$$

$H(0) = 1$

$$H(j1) = \frac{j1 + 1}{-1 + 2j1 + 2} = \frac{1 + j1}{1 + j2}$$

$1 + j1 = \sqrt{2}\, e^{j45}$

$\sqrt{2}$, 1, 1

$1 + j2 = \sqrt{5}\, e^{j\,\mathrm{Tan}^{-1}(\frac{2}{1})}$

$\sqrt{5}$, 2, 1

$$|H(j1)| = \frac{|\text{top}|}{|\text{bottom}|} = \frac{\sqrt{2}}{\sqrt{5}}$$

$$\angle H(j1) = \angle\text{top} - \angle\text{bottom} = 45° - \mathrm{Tan}^{-1}\left(\frac{2}{1}\right)$$

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